Design of a New Hierarchical Structured Peer-to-Peer Network Based On Chinese Remainder Theorem

Bidyut Gupta, Nick Rahimi, Henry Hexmoor, and Koushik Maddali
Department of Computer Science
Southern Illinois University
Carbondale, IL 62901, USA
bidyut, hexmoor, nick, [@cs.siu.edu], koushik@siu.edu

Abstract

In this work, we have presented a new hierarchical non-DHT based architecture for Peer-to-Peer (P2P) networks in which at each level of the hierarchy existing networks are all structured and each such network has the diameter of 1 overlay hop. Such low diameters have immense importance in designing very efficient data lookup algorithms. We shall use a mathematical model based on the Chinese Remainder Theorem (CRT), generally used in cryptography, to define the neighborhood relations among peers to obtain the above-mentioned diameters. To the best of our knowledge, use of CRT in P2P network design is a completely new idea; it does not exist in the literature so far.

Keywords: Structured P2P networks, Chinese Remainder Theorem, Network Diameter

1 Introduction

Peer-to-Peer (P2P) overlay networks are widely used in distributed systems. There are two classes of such networks: unstructured and structured ones. In unstructured systems [1], [2] peers are organized into arbitrary topology. Flooding is usually used for data look up. Problem arising due to frequent peer joining and leaving the system, also known as churn, is handled effectively in unstructured systems. However, it compromises with the efficiency of data query and the much needed flexibility. On the other hand, properly designed structured architectures can offer efficient, flexible, and robust service [3] - [5], [7], [8]. Such overlay networks use distributed hash tables (DHTs) to achieve efficient data insertion, lookup etc. However, maintaining DHTs is a complex task and it needs huge amount of effort to handle the problem of churn. So, the major challenge facing such architecture is how to reduce this amount of effort related to handling of churn while still providing an efficient data query service. There exist several important works, which have considered designing hybrid systems [6]; their objective being to incorporate the advantages of both structured and unstructured architectures. However, these works have their own pros and cons.

Problem formulation: In this work, we have considered interest-based P2P systems [6], [9], [10], [11]. We will consider designing of non-DHT based hierarchical P2P architecture in which at each level of the hierarchy existing networks are all structured and diameter of each such network is 1 overlay hop. Such low diameters will have immense importance in designing very efficient data lookup algorithms. We shall use a mathematical model based on the Chinese Remainder Theorem, generally used in cryptography, to define the neighborhood relations among the peers to obtain the above-mentioned diameters. To the best of our knowledge, it is the first time that neighborhood relations among peers in an overlay network will be defined using such a mathematical model. Besides, to the best of our knowledge, it is also the first time that a successful attempt is made to design structured hierarchical P2P networks with all its component subnetworks having the diameter of 1 overlay hop only.

The paper is organized as follows. In Section 2, we state in detail the proposed architecture and the mathematical foundation used in the design phase. In

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Section 3 we highlight the main features of the architecture. Section 4 draws the conclusion.

2 Proposed Architecture

In this section, we present a structured architecture for interest-based peer-to-peer system [6], [10], [11] and the required mathematical basis supporting the architecture. We use the following notations along with their interpretations while we define the architecture.

We define a resource as a tuple <R_i, V>, where R_i denotes the type of a resource and V is the value of the resource. A resource can have many values. For example, let R_i denote the resource type 'songs' and V' denote a particular singer. Thus <R_i, V'> represents songs (some or all) sung by a particular singer V'. In the proposed model for interest-based P2P systems, we assume that no two peers with the same resource type R_i can have the same tuple; that is, two peers with the same resource type R_i must have tuples <R_i, V'> and <R_i, V''> such that V' $\neq$ V''.

We define the following. Let S be the set of all peers in a peer-to-peer system. Then S = $\{P^{Ri}\}$, $0 \le i \le r-1$. Here P^{Ri} denotes the subset consisting of all peers with the same resource type R_i and no two peers in P^{Ri} have the same value for R_i and the number of distinct resource types present in the system is r. Also for each subset P^{Ri}, P_i is the first peer among the peers in P^{Ri} to join the system.

We now propose the following architecture suitable for interest-based peer-to-peer system. We assume that no peer can have more than one resource type.

2.1 Two Level Hierarchy

We propose a two level overlay architecture and at each level, structured networks of peers exist. It is explained in detail below.

At level 1, we have a network of peers such that peers are directly connected (logically) to each other. In graph theoretic term, the network at level 1 is a complete graph. Hence, the network diameter is 1. The periphery of this network appears as a ring network and we name it as transit ring network. This network consists of the peers P_i ($0 \le i \le r-1$). Therefore, number of peers on the ring is r and we have assumed that this number represents the number of distinct resource types of the P2P system. Each of these r peers will be termed as a group head. The periphery of this network as well as the direct links connecting any two peers in this network can be used for efficient data lookup.

At level-2, there are r numbers of completely connected networks of peers. Each such network, say N_i is formed by the peers of the subset P^{Ri}, ($0 \le i \le r-1$), such that all peers (ϵ P^{Ri}) are directly connected (logically) to each other, resulting in the network diameter of 1. Each such N_i is connected to the transit ring network via the peer P_i, the group-head of network N_i. From now on network N_i will be called as $group_i$ (in short as G_i) with P_i as its group-head. Sometimes N_i will be referred to as the i^{th} cluster as well. The proposed architecture is shown in Figure. 1.

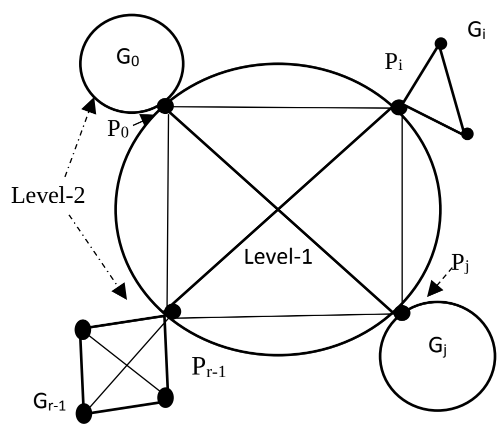

Figure 1: A two-level structured P2P architecture with r distinct resource types

We now state a brief sketch of our approach to realize the architecture. We shall determine a simultaneous solution (a positive integer) of a given system of linear congruencies and then determine some more solutions as needed to form the architecture, which are congruent to the simultaneous solution. For this, we shall take help of the Chinese Remainder Theorem (CRT). Each such solution will become the logical address of a group head uniquely. At the same time, we will determine separately the solutions of each linear congruence as needed and these solutions will represent the logical addresses of the peers present in a group. The following interesting facts will be revealed.

(a) The neighborhood relationships among the group heads based on the logical addresses assigned to them will be shown to form the network at level 1 with diameter 1. In graph theoretic term, each G_i is a complete graph.
(b) Assignment of logical addresses to peers in a subnet P^{Ri} (i.e. group G_i) based on the solutions of an individual linear congruence used in the CRT will guarantee that all peers in G_i are directly connected to each other (logically) forming an overlay network of diameter 1.
(c) Assignment of logical codes to distinct resource types will be the same as the group heads' logical addresses. That is, group head P_i of group G_i possessing resource type R_i will have the same logical address as the code for the resource R_i.

Below we give an overview of Chinese Remainder Theorem, which will offer the mathematical foundation of the proposed architecture.

2.2 Chinese Remainder Theorem

Chinese Remainder Theorem (CRT) states the following:
Suppose $m_0, m_1, m_2, \ldots, m_{r-1}$ are r integers such that no two of which have a common factor other than 1.
Let $M = m_0\, m_1\, m_2 \ldots m_{r-1}$. Also suppose that a_0, a_1, a2, … , a_{r-1} are integers such that gcd $(a_i, m_i) = 1$ for each i.
Then the r congruences given as
$a_i x \equiv b_i \pmod{m_i}$, $0 \le i \le r-1$ have a simultaneous solution that is unique modulo M and is given as

$$x_0 = c_o\, n_1\, n_1^{\sim} + c_2\, n_2\, n_2^{\sim} + \ldots + c_{r-1}\, n_{r-1}\, n_{r-1}^{\sim}$$

where $n_i = M/m_i$ and gcd $(n_i, m_i) = 1$; that is, $n_i\, n_i^{\sim} \equiv 1 \pmod{m_i}$; and c_i is an integer such that $a_i c_i \equiv b_i \pmod{m_i}$, for $0 \le i \le r-1$

An example: $x \equiv 2 \pmod 3$ (1)
$x \equiv 3 \pmod 5$ (2)
$x \equiv 2 \pmod 7$ (3)

These three congruencies satisfy all restrictions of CRT.
We see that $a_1 = a_2 = a_3 = 1$ and $c_1 = 2$, $c_2 = 3$, $c_3 = 2$; M = 105; also $n_1^{\sim} = 2$, $n_2^{\sim} = 1$, $n_3^{\sim} = 1$
Therefore, the simultaneous solution satisfying all these three congruencies is 23 (the least positive solution) and all solutions are congruent to 233 (mod 105). That is, all solutions of the form 23+k.105 (k is an integer) are mutually congruent as well since 'congruence relation' is an equivalence relation.

2.3 A Sample P2P Architecture

Suppose that a P2P architecture is to be built based on CRT with peers that possess one of three distinct resource types, R_1, R_2, R_3, i.e. r = 3. Therefore, we will use CRT to determine a simultaneous solution for three linear congruencies, which is unique modulo M, M being the product of the three moduli used in the congruencies. The three congruencies must satisfy the CRT's requirements. We state first how to assign logical addresses to peers on the transit ring network; note that these peers are the group heads as defined earlier. Also, note that in any structured P2P system, neighborhood relations among peers are defined by the mathematical model used to build the architecture. The mathematical model is intimately related to the efficiency of different data lookup schemes used in a given structured P2P system. We shall use the above example to realize the architecture.

Level 1 address assignment and neighborhood relations: Suppose that P_1 is the first peer to join the system and incidentally with resource type R_1. Therefore, P_1 is the group head of group G_1. We assign the logical address 23 (the least positive solution of the example). Later when other peers with the same resource type join, they are placed in the group G_1. We will explain a little later how to assign their logical addresses and how they are linked. Next, suppose that peer P_2 is the first to join the system with resource type R_2 among all other peers that have the same resource type. So P_2 becomes the group head of group G_2. It will get the logical address as the second least positive solution which is 128 (= 23 + 105). In a similar way, the first peer to join the system with resource type R_3 (say, it is P_3) becomes the group head of group G_3 and gets the next least positive solution 233 (= 128 + 105).

According to the mathematical model, two peers are neighbors if their logical addresses are mutually congruent. Therefore, because of the 'equivalence' property of congruence relation, the logical addresses of P_1, P_2, and P_3 are mutually congruent. Hence, we link P_1 and P_2 directly; it is true for P_2 and P_3; also for P_1 and P_3. Observe that such overlay links among P_1, P_2, and P_3 have created a complete graph of three peers at Level 1. That is, the diameter is 1 overlay hop.

Resource type R_i possessed by peers in G_i is assigned the Level-1 logical address as its code. It will have a very positive impact on the efficiency of data lookup algorithms.

Level 2 address assignment and neighborhood relations: Let us start with the peers of group G_1 since we have assumed earlier that its group head P_1 is the first peer to join the system. We have to use one of the three congruencies as mentioned in the example above. Without any loss of generality, let us consider the first one, viz., $x \equiv 2 \pmod 3$ for the address assignments for peers in G_1. Note that this congruence is a Linear Diophantine Equation (LDE) with one mutually congruent solution. Its least positive solution is 2 and all solutions of the form 2+k.3 (k is an integer) are mutually congruent since 'congruence relation' is an equivalence relation. We assign the addresses 2+k.3 (with increasing values of k) to the peers based on their sequence of arrivals into the system. Therefore, P_1 gets the address 2 (the least positive solution of the congruence) because it arrives before any other peer with resource type R_1; the second arriving peer will have the address (2+3), the third peer will have the address (2+2.3) and so on. As before, based on our mathematical model two peers are neighbors if their logical addresses are mutually congruent; meaning thereby that two peers in a group are directly linked if their logical addresses are mutually congruent. So because of the equivalence relation, each peer in G_1 is logically connected to every other; hence, the diameter of the group G_1 (i.e. the cluster N_1) is 1 overlay hop, making the cluster a complete graph.

In a similar way, the second congruence is used to assign the addresses to P_2 and the other members in this group G_2 (cluster N_2). Thus P_2 gets the address 3 and 3+k.5 are assigned to others in the cluster based on their sequence of arrivals. As before, this cluster is also a complete graph.

Similarly, the third congruence is used to assign the addresses to P_3 and the other members in this group G_3 (cluster N_3). Thus, P_3 gets the address 2; and 2+k.7 are assigned to others in the cluster based on their sequence of arrivals. This cluster is also a complete graph with diameter of 1 overlay hop. The architecture is shown in Figure. 2. In this example, we assume that the groups G_1, G_2, and G_3 consist of 2, 3, and 4 members respectively.

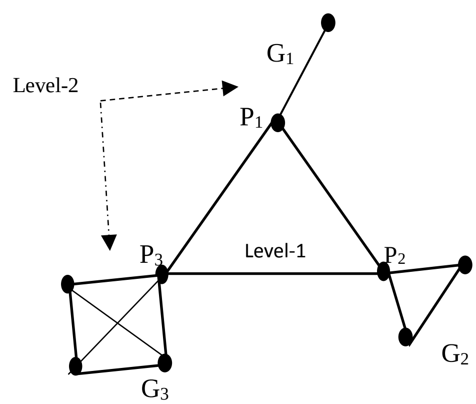

Figure 2: A Structured P2P architecture with 3 distinct resource types

Observation 1*: Each group head has two different logical addresses; one from Level-1 assignment and one from Level 2 assignment.*

It has important implications related to intra-cluster and inter-cluster data lookup queries. Level 1 assigned address is used to move the query from one cluster to another for inter cluster sequential lookup; Level 2 assigned address is used to answer the intra-cluster query.

Observation 2*: Different group heads may get identical Level 2 assigned addresses.*

It will not affect any intra-cluster lookup query in a cluster, as this address is local to this cluster only.

3 Salient Features of the Overlay Architecture

We summarize below the salient features of the proposed architecture.

1. It is a hierarchical overlay network architecture consisting of two levels; at each level, the network is a structured one.
2. Use of Chinese Remainder Theorem (CRT) helps in the construction of the Level-1 network as a complete graph, i.e. its diameter is 1 overlay hop. The implication is imminent in that it will take just one overlay hop to move from one cluster (group) to another cluster.
3. Use of (CRT) allows a group-head address to be identical to the code given to a resource type owned by the group. It helps in moving a data lookup query from one cluster to another for inter cluster sequential lookup.
4. Number of peers at Level-1 is the number of distinct resource types, unlike in existing distributed hash table-based works some of which use a ring network at the heart of their proposed architecture [5], [6].
5. Use of LDE makes each overlay cluster at Level-2 a completely connected network. That is, like Level-1, the diameter of each Level-2 cluster is also 1 overlay hop.

4 Conclusion

In this work, we have presented a novel two level hierarchical P2P network architecture in which diameter of each cluster at Level- 2 is just 1 overlay hop and diameter of the network at Level-1 is also 1 overlay hop. This is a significant improvement over our recently designed highly efficient two level hierarchical architecture [9], [10], [11]. Such low diameters will have immense importance in designing very efficient data lookup algorithms. Building such an architecture has been possible because of the use of a mathematical model based on the Chinese Remainder Theorem, which is used in cryptography. To the best of our knowledge it is the first time that neighborhood relations among peers in an overlay network has been defined using such a mathematical model. We have pointed out some important architectural characteristics that can be fetched using the CRT. 'Work in progress' includes design of low latency data lookup algorithms, anonymity consideration in communication protocols, and design of effective churn handling protocols.

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