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# Modeling the Effects of Network Games on Social Reasoning

**Ivan Belik[1], Henry Hexmoor[2]**
[1]Norwegian School of Economics, Department of Business and Management Science,
Norway, ivan.belik@nhh.no
[2]Southern Illinois University Carbondale, Department of Computer Science,
USA, hexmoor@cs.siu.edu

**ABSTRACT**

The analysis of social reasoning is at the core of understanding how to manage social networks. Since interpersonal relations are composed of multiple factors with different nature (i.e., structural and social factors), we explore their influence on the strategizing processes in social networks. The research is based on the consideration of social networks in terms of network games. Therefore, we formalize interpersonal relations using the methods of structural and social analysis combined with game theoretic approach. Specifically, we formalize social power of an agent and use it to quantify payoffs. Based on reasoning over values of power we show how individuals reach stability and Nash equilibrium with their peers in network games.

**Key words:** agent's power, social networks, network games

## 1. INTRODUCTION

The framework of the research is based on the problem of modeling the effects of network games on social reasoning.
A social network is considered as an n-person nonzero sum game. Basically, each agent is characterized by structural metrics (i.e., centralities) and by social characteristics, such as measure of trust to other players. In fact, the research corresponds to the investigation of functional dependencies between the logical and mathematical apparatuses of three interconnected concepts described next.

### 1.1 Structural analysis of social networks

As was mentioned, structural analysis is a basic component of the investigation process. We use three fundamental structural measures in the given research: (a) degree-based centrality, (b) betweenness centrality and (c) closeness centrality [1]. All of these measures are the components of social power analysis. One of the goals for this research is to encapsulate structural centralities in a unified structural measure. This encapsulation is the first step in the formalization of social power.

### 1.2 Analysis of social networks as the networks of trust

We consider trust as a social property of interpersonal relations in networks. In fact, social networks are based on the exchange of trust between their members (i.e., agents). Trust is at the core of the decision making process of each agent in a social network [2]. The conception of trust can be used in combination with Bayesian networks. The approach is based on the method of Bayesian inference [3].

### 1.3 Formalization of social networks in terms of game theory

Since game theoretic methodologies are well adapted for socio-economic modeling, the representation of a social network in a game form can provide beneficial effects [4]. Game theoretic methods can be used for the formalization of agents and their relations in social networks. Each agent can be considered as a player and agent's benefits or losses can be represented as a player's payoffs. In fact, each player attempts to maximize or minimize its payoff by strategizing [5]. Since each player chooses the extent of trust to another player, we consider a player's choice of the level of trust as a strategy. Application of game theory in social networks optimizes the balance of interaction among individuals. For example, game theory provides a network with stability or equilibrium [5].

The main idea of the research is to investigate and mathematically formalize interdependencies among structural and social factors. Furthermore, the main challenge is to adapt this formalization for the game theoretic representation and analysis of gaining stability and Nash Equilibrium for the network game.

## 2. BACKGROUND

The analysis of social networks is basically related to their structural analysis. One of the first structural models based on the theory of directed graphs was suggested by [6]. It includes basic mathematical formalization and explanation of graph theoretic methodologies and their application in formalization of networks. Theory of directed graphs is a mathematical formalization of networks that can be applied to any types of networks represented by graphs (i.e., not only social networks). The theory of directed graphs is closely related to power networks [7]. According to [7], power is an agent's ability to influence other agents and to resist an influence from other agents in the network. The computation of structural measures is considered as a basic step of the analysis of social networks. Harary's research is concentrated on the investigation of social properties of agents, such as "power", "dominance", "dependence" and "status". Power networks are based not only on the structural analysis of networks, but also on the formalization of social interrelations among agents. This approach is widely used in the analysis of social systems, such as exchange networks [1].

Exchange networks are socio-economic networks that can be characterized by five properties [1]. First, an exchange network is a set of agents and interrelations between them. Second, network resources are distributed between agents. Third, each agent makes a decision regarding the exchange process according to its individual interests. Fourth, each agent has a personal history of exchange within a network. Fifth and last, all interpersonal relations are encapsulated in a unified exchange network. According to [1], the formalization of exchange networks is based on two basic aspects: structural analysis and internal power of relations. Specifically, Cook et al. [1] used three basic measures for the structural analysis: (a) degree-based centrality, (b) closeness-based centrality, and (c) betweenness-based centrality. The analysis of internal power includes two factors: power and dependence. Power is considered as an agent's potential to obtain the desired outcome from other agents in the network. Dependence implies the separability of opportunities and limitations of power distribution between different agents. It means that the relation between agent A and agent B is characterized by the dependence that is different from the dependence between agent A and agent C. According to [1], structural measures and internal power of relations are interdependent and influence each other.

Social networks can be analyzed from the different angles. According to [8], efficiency is one of the most important properties of social networks. Reference [8] described the efficiency of social and economic networks in three basic categories. The first is that the notion of efficiency is the Pareto efficiency. Pareto efficiency (i.e., Pareto optimality) is a specific state of social network when an improvement of an agent's condition is impossible without worsening the conditions of other agents. Pareto optimality is based on the idea that all profits from the operations of exchange within a network are exhausted. It means that if at least one agent starts to improve its condition, then it will change the state of another agent or agents in a negative way.

According to [8], an agent is a member of the Pareto efficient network if there is no other network that can guarantee a better benefit than the current network. The second definition of efficiency is related to the maximization of an agent's benefit [8]. It does not mean that each agent will maximize its payoff. The basic idea of such kind of efficiency is that the total amount of all payoffs should be maximized. The third conception of network efficiency is related to the availability of specific types of transactions for each agent. It means that social network is efficient if the availability to realize the specific set of transactions at any time is guaranteed to each agent. This type of network efficiency implies that agents should not be limited in the realization of the specific set of rights. For example, if any democratic society is considered as efficient, then it should guarantee the freedom of choice and freedom of action for each member. The advantage of the given research is that it includes a deep analysis of the specific models of social networks. For example, Jackson (2003) considered the Connections Model [9] and the Co-Author Model [9].

Another approach regarding the network power and structural measures was done by [10]. The research is based on the abstract formalization of interdependencies between an agent's power and centrality. Reference [10] did not specify which structural measures are better to be used for the structural analysis of social networks. It considered bargaining situations where agent's power is its bargaining power. According to [10], it is preferable for an agent to keep relations with agents who have less bargaining power. If agent A keeps relations with more powerful agents, then it will have less influence in the bargaining process. This implies the decrease of the bargaining power for agent A. Reference [10] analyzed the problem of interrelations between network properties conceptually without specific computations. Mathematical formalization of interrelations between structural measures is abstracted away from the use of specific measures.

## 3. METHODOLOGY

A social network is a network that has a specific topology and social structure. The basic objects of social networks are agents (i.e., individuals, companies, and communities) that are represented by nodes and related by different kinds of social relations (i.e., friendship, love, trust, business, and knowledge). In fact, a social network can be represented as a graph as shown in Figure 1. Every social network can be analyzed by graph theoretic methodologies. Social networks have different structural complexity, but in practice, they are considered as large-scale networks. This is due to the fact that they mimic the complexity of real-world social interdependencies.

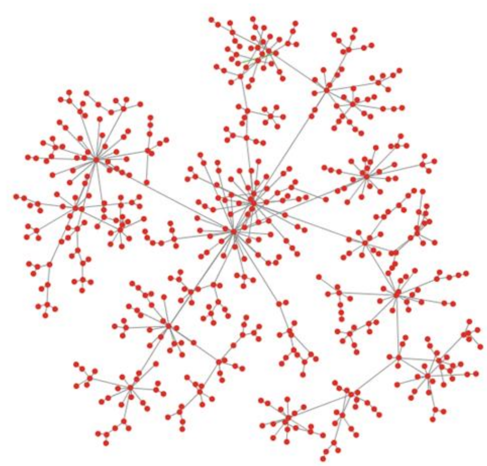

**Figure 1:** A Prototypical Social Network with a Mixed Topology

Quantification of social power is a multifactor analysis of the agent's role in any kind of social and economic network. It is strongly related to the level of agent's influence on each member of the network and on the integrity of the network. To put it more simply, social power captures a level of an agent's importance and an agent's opportunities within a social network.

Social power can be characterized by many measures. For example, Cook et al. [1] used structural centrality as a primary factor for social power. They used three basic measures of (a) degree-based measure, (b) betweenness measure, and (c) closeness-based measure in order to compute the distribution of power in exchange networks. Brandes&Pich [11] used two measures of (a) closeness and (b) betweenness for centrality estimation in large networks. Another important factor of social power is an agent's internal power, which characterizes an agent's resources (i.e., energy, knowledge, and trust).

Social power structure is represented in Figure 2. Next, we describe the components in detail.

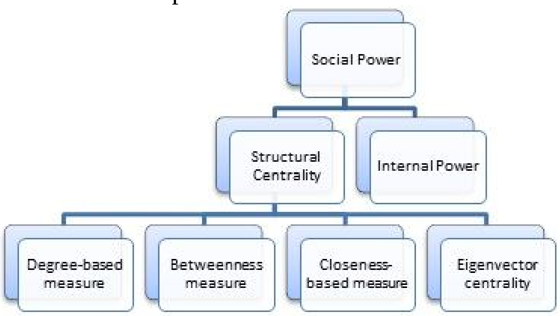


**Figure 2:**Social Power Structure

### 3.1. Structural Centrality

Structural centrality is the most important concept in social power. It is based on the structural analysis of networks. Every social network can be represented as a graph. Formalization of structural centrality is closely related to the mathematical approach in graph theory. It is based on the computation of the shortest-path distances in the graphs, frequencies of nodes on the shortest paths, and connections of vertices to the low/high scoring nodes. Structural centrality is a measure of an agent's importance in terms of the structural analysis of networks.

#### 3.1.1 Degree-based measure (degree centrality)

Degree centrality (DC) of a vertex is a number of links directly connected to it. According to [12], DC of a vertex can be characterized as an indicator of its potentiality to interact with other vertices.
Based on [9], DC computation for a vertex $v$ of a graph G(V, E) with n nodes can be realized by (1).

$$DC(v) = \frac{\deg(v)}{n-1} \quad (1)$$

where$\deg(v)$ is a number of nodes directly connected to $v$.

#### 3.1.2 Betweenness measure (betweenness centrality)

Betweenness centrality (BC), as the measure of structural centrality, estimates how often the particular vertex can be visited looking through the shortest paths between all possible pairs of vertices [12].
Equation (2) represents BC computation [13],[14]:

$$\mathrm{BC(v)} = \frac{\sum_{s \neq v \neq t} \sigma(s,t|v)}{\sigma(s,t)} \quad (2)$$

where:
σ(s,t) is the number of the shortest paths among all paths from s to t;
σ(s,t|v) is the number of the shortest paths starting at s, visiting v and ending in t.

#### 3.1.3 Closeness-based measure (closeness centrality)

Closeness centrality (CC) measures how close the given vertex is to all other vertices of the graph on average. An agent with the highest closeness can be approached from elsewhere in the network faster on average than any other agent. CC has an important practical use because it allows for determining the best position in the network from which other agents can be easily reached.

CC is inversely related to the sum of the shortest distances from vertex $v$ to all other nodes [15],[16]. Distance is considered as a number of edges in the shortest path between two vertices.

$$CC(v) = \frac{1}{\sum_{t \epsilon V \backslash v} d_G(v,t)} \quad (3)$$

where$d_G(v,t)$is the shortest distance between vertices 'v' and 't' in graph G.
Equation (3) works well only with connected graphs. The modification of this formula was offered by [17]:

$$CC(v) = \sum_{t \epsilon V \backslash v} 2^{-d(v,t)} \quad (4)$$

Equation (4) is adapted to work with disconnected graphs.

#### 3.1.4 Eigenvector centrality

Eigenvector centrality (EC) measures an agent's significance with respect to other agents in the network. It characterizes quantitative and qualitative performance capabilities of agents [18]. In other words, more powerful agents can be more beneficial, and it is preferable to keep connections with them.

According to [18], EC of the agent *i*is proportional to the average total EC score of its neighbors:

$$x_i = \frac{1}{\lambda} \sum_{j=1}^{n} A_{ij}\, x_j \quad (5)$$

Here:
$A_{ij}$ is a network's adjacency matrix. If vertex $i$ is directly connected to vertex $j$, then $A_{ij} = 1$; otherwise, $A_{ij} = 0$;
**λ** is a constant.

Some EC values for nodes are a priori known. Since equation 5 is recursive, the a priori values seed initial values used to compute values of EC for other agents.
Alternatively,(5) can be represented in matrix form [18]:

$$\lambda x = A \cdot x \quad (6)$$

Here:
x is an eigenvector of centralities;
λ is an eigenvalue of matrix A.

### 3.2. Internal Power (IP)

IP is the second approach for social power quantification. It characterizes the internal agent's resources. Compared to structural centralities, IP is not related to the structural features of the network, but it works with the internal characteristics of connections between agents. The specification of IP depends on the area of its application. For example, in terms of economics agent's IP can be represented by capital, money, investments, and other tangible quantities. Current research focuses on the social foundation of agent's IP. Accordingly, we characterize IP by three internal components: energy, knowledge and trust.

#### 3.2.1 Energy

Energy is an abstraction of social and economic resources. One of the interpretations of energy as a social category is given by [19]. In the context of social analysis, energy can be represented by an agent's ambitions, willpower, and social activities. In terms of economic analysis, energy can be represented by money, time, and propensity for financial risk. Both kinds of energy are limited. For example, an agent cannot work more than 24 hours per day or spend more money than it has. An aggregated agent's energy can be represented by any value in the range [0, 1].

#### 3.2.2 Knowledge

Knowledge is what is known by an agent regarding its position in the network. It includes the information regarding the states of other agents, connections, and network characteristics in general. In the context of social power, knowledge can be characterized as the level of an agent's information awareness about the network. The deep analysis of knowledge as a social category is done by [20].

#### 3.2.3 Trust

Trust is a basic characteristic of social networks. If agent A does not trust agent B, then agent B will not get any benefit from agent A, which includes energy and knowledge.We consider a trust network as a directed graph, where trust can take on any value from a range [0, 1]. Therefore, a mathematical apparatus applied for directed graphs can also be used in trust networks. One of the interesting interpretations of trust is given by [2], where trust is considered as an abstract and personal category of interpersonal relations.

It is important to say that social power has already become one of the most important parameters in the analysis of social and economic networks. It is not just an abstract and uncertain philosophic term, but it is a deeply formalized concept of mathematical formalization in social and economic networks.

### 3.3 Network Games

Network games are a combined approach that is based on the structural analysis of socio-economic networks and game theory. Players and their interrelations form a network that can be represented as a graph. Network games are based on the extended analysis of players' interdependencies. Such approach implies the symbiosis of graph theory and game theory methodologies for the purpose of more adequate analysis of socio-economic relations. According to [4], network game investigations are based on four basic factors: player's degree $d$, player's probabilistic property $\sigma(d)$, utility function $u_d$, and agent's strategy $x_i$.

A player's degree is a structural measure. It corresponds to interdependencies between agents in terms of graph theory. A player's degree is an estimated number of an agent's relations with other agents. In fact, it corresponds to the number of directly connected nodes of graph, where each node is a player. An agent's property $\sigma(d)$ is a probabilistic characteristic of an agent $i$ with degree $d$. It is a probability that a neighbor of agent $i$will choose a specific strategy [4]. The values of $\sigma(d)$ are in the range [0, 1]. According to [4], agent's utility $u_d$is its payoff based on the current degree $d$, probabilistic property $\sigma(d)$ and the chosen strategy $x_i$. Reference [4] considers network games in terms of manipulations with these three basic factors (i.e., $d$,$u_d$,and $\sigma(d)$) for the purposes of network stabilization and Nash equilibrium gaining.

Another network games analysis is done by [21]. It is based on the idea that a player's payoff depends on the structural state of its neighbors. Reference [21] also considered network games in terms of combined methodologies of structural analysis and game theory. Reference [21] gave a mathematical interpretation of such kinds of interrelated methodologies. They consider network games with incomplete information about network structure. The formalization of agents' payoffs is based on the correlated analysis of three basic aspects: (a) structural centralities, (b) information incompleteness, and (c) network externalities.

## 4. APPROACH

### 4.1 Formalization of Social Power

Measures that characterize social networks are often motivated independently. For example, centrality and density are heterogeneous measures of a social network and cannot be easily combined since they quantify measures of interest for different uses of social networks.

Structural network analysis attempts to understand the internode connectedness as in graph theory methodologies. The analysis of different types of structural measures in terms of social networks was done by [22]. Graph based network methodologies cannot be applied for analysis of social factors in social network processing because social networks possess social content that cannot be reduced to measurement by structure. In contrast to structural analysis, social analysis has a different foundation and cannot be quantified by topological analysis.

### 4.2 Structural Centrality and Trust

Three measures of structural centrality are taken into consideration: (a) degree-based measure, (b) betweenness measure, and (c) closeness-based measure. To accomplish interdependency, these three measures are unified in one structural parameter that is called structural centrality (SC). A problem is that each measure takes its values from different numerical intervals. The process of unification is based on the idea that SC should take its value from a unified interval, say [0, 1].

The method of unification for structural parameters is based on the knowledge about the minimum and maximum values of each parameter at the particular moment (i.e., snapshot of the network). Each agent is characterized by values of three structural measures mentioned, and each structural measure may have any value greater than or equal to zero at a particular moment (i.e., at a snapshot) of the network game. The agent with the minimum value of the particular structural measure will set the lowest value of this structural measure corresponding to "0" value equivalent in the range [0, 1]. Accordingly, the agent with the maximum value of the considered structural measure will set the highest value "1" in the range[0, 1]. For example, let's consider the betweenness centrality (i.e., measure) in the trivial network consisting of three agents shown in Figure 3. Numbers inside nodes represent centrality values.

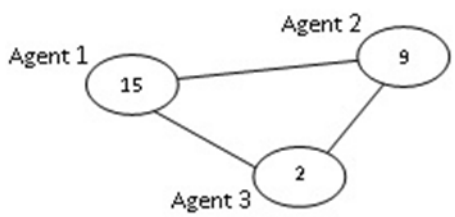


**Figure 3:** A Trivial Network Example with Betweenness Centrality Values

According to Figure 3, agent 1 has a maximum betweenness centrality value of 15. This means that value of 15 will be mapped to 1 in the range [0, 1]. Agent 3 has a minimum betweenness centrality value of 2. Value of 2 will be mapped to 0 in the range [0, 1].
Having upper- and lower- bounds of the betweenness centrality, all other intermediate values can be computed in the interval [0, 1]. Particularly, agent 2 will have betweenness centrality value interpolated to 0.69.
The methodology we described above is applied for all three measures taken into consideration. The unified value of social centrality (SC) is determined by (7).

$$SC = \frac{DC + BC + CC}{3} \quad (7)$$

where $SC \in [0, 1]$.
Equation 7 is founded on the idea that all structural measures contribute equally to the general SC. Our formulation specifies a linear composition between them. A linear composition is stipulated by structurally equal importance of degree-based, betweenness and closeness-based measures for an agent's structural centrality. Structural analysis is at the core of each centrality measure, but the difference is that each measure is based on the consideration of network structure from a specific angle. Each of these measures is a quantitative characteristic of an agent's structural centrality. An arithmetic mean computation (i.e., (7)) is a method to avoid the prioritizing of their contributions to a general agent's structural centrality. In fact, the consideration of non-linear composition implies different levels of structural measures' importance. In this case, each structural measure should have some specific characteristics (excepting structural) to be considered as a more or less important measure. A good example is an eigenvector centrality (5) that is not only quantitative, but also qualitative structural measure. Equation 7 cannot have a linear composition if it includes an eigenvector centrality. Nevertheless, an eigenvector centrality is not used in (7).

Once we consider a network that represents social nature of interactions, we can interpret such a network to be a network of trust [3], [23]-[25]. Basically, agents can measure trust and represent values in the range [0, 1]. An agent lacks trust at all (i.e., the fewest trust) or has an abundant trust (i.e., the most trust) to another agent if the values of trust are equal to 0 and 1 respectively.

### 4.3. Formalization of Social Power

Having unified values of structural measures and trust, it is necessary to amalgamate them into a single function:

$$Y = f\,(SC, T) \quad (8)$$

One of the basic analyses of interdependencies between structural centrality and trust was done by [26]. Reference [26] investigated the interdependencies between two components of social networks: structural measures and trust. The functional dependencies were formalized for the relations between buyers and sellers. The conception of(8) is another point of view for the interpretation among social network relations. It is not limited by the consideration of specific socio-economic interactions, because it is based on the conceptual analysis of social relations.

The proposed idea in this research is to consider equation 8 as the combined social power of agent A (see Figure 4). According to Figure 4, social power of agent A (i.e., computed using(8)) depends not only on the current structural centrality of agent A and its trust (T) with respect to other agents (namely B and C in Figure 4), but also on the current structural centralities of the other agents and their trust on agent A.

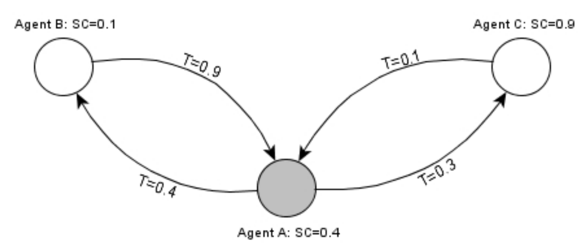


**Figure 4:** A Trivial Example of Network with Trust and Social Centrality Relations

In fact, the combination of T and SC can be termed as an agent's social centrality or social power (SP). Equation 9 elaborates (8).

$$SP_A = \frac{\sum_{i=1}^{N-1} T_{i,A}}{N-1} \times SC_A + \frac{\sum_{i=1}^{N-1}(T_{A,i} \times SC_i - T_{i,A} \times SC_A)}{N-1} \quad (9)$$

Here,
N is a number of agents;
$T_{i,A}$ is a trust from agent *i* to agent A;
$T_{A,i}$ is a trust from agent A to agent *i*;
$SC_A$ is a structural centrality of agent A;
$SC_i$ is a structural centrality of agent *i*.

Equation 9 consists of two main components.

1. $\left(\frac{\sum_{i=1}^{N-1} T_{i,A}}{N-1} \times SC_A\right)$.

This encapsulates the basic interdependency between SC for agent A and T to agent A from all other agents. Agent A may have the highest SC in the network. However, if no one trusts it, A will not experience any social power.
$\left(\frac{\sum_{i=1}^{N-1} T_{i,A}}{N-1}\right)$ computes an average T from all agents at the network toward agent A.

2. $\left(\frac{\sum_{i=1}^{N-1}(T_{A,i} \times SC_i - T_{i,A} \times SC_A)}{N-1}\right)$.

Social power of agent A can be consistent with the influences from all other agents. I.e., current structural centrality of the other agents and their levels of trust to agent A. This influence makes social power more sensitive to feedback from other agents and their current conditions compared with the current individual outcomes from agent A to each agent. This component can take on a positive or negative value.

Social power is the formalization of functional interdependencies between attributes that have different nature (i.e., structural vs. social). For example, if agent A is in the same structural condition (i.e., SP=1) as all other agents and its trust to other agents is at maximum level, then agent A possess the biggest social power in the network even if all other agents do not trust agent A at all (i.e., $\sum_{i=1}^{N-1} T_{i,A} = 0$).

It is important to notice that the given model of social power can be augmented by the extended considerations of social factors. If any other social relations (e.g., knowledge, friendship, and love) can be measured numerically, unified to the range [0,1] and represented by functional interdependency, defined by Z=(social factor 1,…, social factor N), then T in (11) can be replaced by Z. It means that T in (9) can be replaced by a multi-factor model of encapsulated social factors like it is done by the implementation of SC multi-factor model in (7) for structural factors.
The main limitation here is that many social factors cannot be easily measured numerically. This replacement possibility shows that the proposed SP-function is flexible for multi-factor analysis of social networks and can be operated with different social and structural parameters without radical change.

**5. GAME THEORETIC APPROACH**

The analysis of social network games in terms of game theory is based on the idea of finding stability or equilibrium for combinations of players' strategies. In terms of given research, stability is considered as the state of network games when the specific combination of player's strategies can ensure a certain condition of the network game. Equilibrium is considered as Nash equilibrium.

The proposed work is based on the idea to consider n-person nonzero sum games. These games can be divided into two types. The first type is the games with deterministic payoffs, where agent's payoff is a value of social power, and strategy is a value of trust. The second type is games with probabilistic payoffs, where agent's payoff is a probability to get a specific value of social power and strategy is a value trust.

**5.1 Network Games with Deterministic Payoffs**

Two basic concepts of the game that have to be determined are agents' strategies ($q=\{q_1, q_2, \dots, q_n\}$) and agents' payoffs ($u_i(q)$, where i=1, …, N).
In the context of the given research, trust is considered as a strategy. A player's strategy construction is a choice process of how to trust other agents. Each agent can choose the level of trust from the range [0, 1]. The given freedom of social choice (i.e., freedom to choose the level of trust) is at the core of strategizing.

An agent's payoff is its social power, and it can be represented by (10):

$$u_A = f\,(q_{A,i}, \dots, q_{A,N-1}, q_{i,A}, \dots, q_{N-1,A}), \quad (10)$$

where *i* is the number of agents that are different from agent A (i.e., *i* = 1, …, N-1).

We consider $u_A$ as a function of trusts from agent A to all other agents and from all other agents to agent A.
An agent's payoff is equal to value of its SP specified in (9):

$$u_A = SP_A \quad (11)$$

The fact that social power is an equation of two variables (i.e., T and SC) is not an issue in the context of the proposed formalization of social network games, because a player can choose only the level of trust during the game, but not the value of structural centrality. Structural centrality is a structural measure and it does not depend on the player's strategizing. A player cannot choose its structural centrality. Since it is a network structural characteristic it becomes a parameter in the payoff function and its value can vary from one game to another. A player can manipulate only by its social choice (i.e., level of trust), and this manipulation affects social power and structural centrality because of their functional interdependency (i.e., SP=$f$(T,SC)).

According to [5], Nash equilibrium ($q_1^*, q_2^*, \dots, q_n^*$) for n-person nonzero sum games can be gained if three conditions are simultaneously satisfied:

1. $\frac{\partial u_i(q_{i,\dots,}q_n)}{\partial q_i} = 0, i = 1, 2, \dots, n.$

2. $q \mapsto u_i(q_1^*, \dots, q_{i-1}^*, q, q_{i+1}^*, \dots, q_n^*) for\ q \in Q_i.$

3. $\frac{\partial^2 u_i(q_{i,\dots,}q_n)}{\partial^2 q_i} < 0, i = 1, 2, \dots, n.$

**Figure 5:** The Conditions for Nash Equilibrium Gaining

The calculus methodology in Figure 5 can be applied for the social network games with deterministic payoffs. We consider the game with two players for the simplification of the Nash equilibrium computation. The methodology of Nash equilibrium gaining is scalable, and the decision for two players can be applied for n-players.

There is a two-person non-zero sum game, where the social power specified in (9) is applied for the payoff's computation and Nash equilibrium gaining.

| **Player 1** | **Player 2** |
|---|---|
| $u_1(q_1, q_2)=q_2\times SC_1+(q_1\times SC_2 - q_2\times SC_1)$ | $u_2(q_1, q_2) = q_1\times SC_2+(q_2\times SC_1 - q_1\times SC_2)$ |
| Verification of (a)-condition: | Verification of (a)-condition: |
| $\frac{\partial u_1}{\partial q_1} = 0 + SC_2 - 0;$ | $\frac{\partial u_2}{\partial q_2} = 0 + SC_1 - 0;$ |
| According to (a) and (b)-condition, the solution cannot be approached by the method of derivation. $SC_2 = 0$ (if (a)-condition is applied) | According to (a) and (b)-condition, the solution cannot be approached by the method of derivation. $SC_1 = 0$ (if (a)-condition is applied) |
| Verification of (c)-condition: | Verification of (c)-condition: |
| $\frac{\partial^2 u_1}{\partial^2 q_1} = 0 \Rightarrow$ (c)-condition is not satisfied. | $\frac{\partial^2 u_2}{\partial^2 q_2} = 0 \Rightarrow$ (c)-condition is not satisfied. |

**Figure 6:** Nash Equilibrium Computations for a Two-Person Non-Zero Sum Game with Deterministic Payoffs

According to Figure 6, n-person non-zero sum games based on (9) can gain the state of stability. A network game stands in a stable condition if all players have the same level of trust to each other:

$$q_1=q_2=\dots=q_n, \tag{12}$$

where "n" is a number of players.

The desired aim of the research to gain the stability of social network games is reached.

Nevertheless, the conditions of the method of derivation (Figure 5) are not satisfied. It means that Nash equilibrium is not gained. However, according to [5], the conditions in Figure 5 are sufficient, but, in practice, they are not necessary. Nash equilibrium can be gained even if the conditions in Figure 5 are not satisfied. Therefore, Nash equilibrium can exist and can be gained by a different approach that is based on n-person non-zero sum games with probabilistic payoffs.

**5.2 Network Games with Probabilistic Payoffs**

As already mentioned, network games can be formalized in terms of probability to get a desired agent's payoff. The formalization of two-person non-zero sum game is represented in (13).

$$u(q_1, q_2) = \begin{cases} Prob\ (SP_1 > SP_2) \\ Prob\ (SP_1 = SP_2) \\ Prob\ (SP_1 < SP_2) \end{cases} \tag{13}$$

To gain the Nash equilibrium for the game formalized by (13), it is necessary to consider all conditions of trust between players and the differences in structural centralities. The consideration of these two factors is required because social power is a function of trust and structural centrality ($SP = f\ (T, SC)$).

The summarized results of computations for all possible states of strategies (i.e., level of trust) and structural centralities are represented in Table 1.

**Table 1:**Combination of Strategies and SCs and Their Effect on SP

| | | | | |
|---|---|---|---|---|
| 1. | **IF $q_1 > q_2$** | | | |
| 1.1 | | AND $SC_1=SC_2$ | THEN | $SP_1>SP_2$ |
| 1.2 | | AND $SC_1<SC_2$ | THEN | $SP_1>SP_2$ |
| 1.3 | | AND $SC_1>SC_2$ | THEN | $SP_1>SP_2$ |
| Thus, | IF $q_1>q_2$ | THEN $SP_1>SP_2$ | | |
| | IF $q_1>q_2$ | THEN Prob $(SP_1>SP_2) = 1$ | | |
| 2. | **IF $q_1 < q_2$** | | | |
| 2.1 | | AND$SC_1=SC_2$ | THEN | $SP_1<SP_2$ |
| 2.2 | | AND $SC_1<SC_2$ | THEN | $SP_1<SP_2$ |
| 2.3 | | AND $SC_1>SC_2$ | THEN | $SP_1<SP_2$ |
| Thus, | IF $q_1<q_2$ | THEN $SP_1<SP_2$ | | |
| | IF $q_1<q_2$ | THEN Prob $(SP_1<SP_2) = 1$ | | |
| 3. | **IF $q_1 = q_2$** | | | |
| 3.1 | | AND $SC_1=SC_2$ | THEN | $SP_1= SP_2$ |
| 3.2 | | AND $SC_1<SC_2$ | THEN | $SP_1= SP_2$ |
| 3.3 | | AND $SC_1>SC_2$ | THEN | $SP_1= SP_2$ |
| Thus, | IF $q_1= q_2$ | THEN $SP_1= SP_2$ | | |
| | IF $q_1= q_2$ | THEN Prob $(SP_1= SP_2) = 1$ | | |

For the purpose of Nash equilibrium gaining, it is necessary to consider a probability of strategy to be chosen by a player. For the given two-person probabilistic game, $\gamma_i$ is a probability that the chosen level of trust from player *i* to another player is greater than the level of trust from another player to the current player *i* (i.e., Prob($q_1 > q_2$) or Prob($q_1 < q_2$)). According to the results of computations represented in Table 1, the Nash equilibrium for this game is ($\gamma_1^*, \gamma_2^*$).

$$(\gamma_1^*, \gamma_2^*) = (\tfrac{1}{3}, \tfrac{1}{3}) \tag{14}$$

Nash equilibrium can also be gained if $\delta_i$ is a probability that the chosen level of trust from player *i* to another agent is greater or equal to the level of trust from another player to the current player *i* (i.e., Prob($q_1 \geq q_2$) or Prob($q_1 \leq q_2$)). According to the results of computations represented in Table 1, the Nash equilibrium for this game is ($\delta_1^*, \delta_2^*$).

$$(\delta_1^*, \delta_2^*) = (\tfrac{1}{2}, \tfrac{1}{2}) \tag{15}$$

Nash equilibrium for social network games with probabilistic payoffs is gained. Also, it is important to notice that structural changes of such kinds of games, based on the SP-function, do not affect Nash equilibrium. For instance, $q_1>q_2$ can guarantee that $SP_1>SP_2$ even if $SC_1< SC_2$. This fact is a proof that Nash equilibrium is gained for dynamic social network games.

## 6. CONCLUSIONS

The analysis of social systems is based on the interdisciplinary approach. It includes not only the social analysis of interpersonal relations, but also game theoretic and graph theoretic methods and concepts. The major aim of the given research was to make a contribution to the understanding of mechanisms of social reasoning using network games as a basic tool. The research begins with a theoretic review of methods and concepts of structural analysis of social networks and game theory, which are required for understanding theoretical methods of the research. The analysis of the related works is done for the understanding of the approaches that contributed expressly or by implication to the modeling of the mechanisms of social reasoning.

The first idea of the research was to combine structural and social properties of agents in a single parameter (i.e., social power). It was approached by the unification of three basic structural measures of social networks, and trust as the basic measure of interpersonal exchange. In fact, the formalization of social power as a multi-factor model of an agent's importance, and authority in a social network gave an opportunity to manipulate social networks using game theoretic methods. Specifically, we considered dynamic social networks as the sequences of network games, where players' payoffs are represented by the values of social power, and strategies are represented by the levels of trust to other agents. The formalization of network games based on the invented equation of social power made it possible to approach the stability of network games with deterministic payoffs. Furthermore, we approached Nash equilibrium of network games with probabilistic payoffs. It implies the practical importance of the research, because the given theoretical approach can be applied in the monitoring and control of real socio-economic systems.

The future work is related to the improvement of the equation of social power. Since social power is a multi-factor model of an agent's capabilities within a network, its current components can be modified and new components can be added. Specifically, we considered trust as a basic social factor of interpersonal relations. However, the other factors, such as knowledge, friendship, and love, can be added to the model if they can be measured. Another improvement may be done in the analysis of network games with deterministic payoffs. We approached the stability of this kind of games, but another challenge is to gain Nash equilibrium.

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