Stochastic Game (Shapley, 1953)
\[ \langle Q, N, A, P, R \rangle \]

\( Q = \) a finite set of strategic games

\( N = \) a finite set of players

\( A = A_1 \times A_2 \times \ldots \times A_n \), \( A_i \) is a finite set of actions for agent \( i \)

\( P = Q \times A \times Q \rightarrow [0,1] \) Transition probability functions

\( P(q, a, q') = \) Probability of moving from \( q \) to \( q' \) on \( a \)

\( R = r_1, \ldots, r_n \) Payoff functions, \( r_i \) is payoff function for Player \( i \)

Strategy Space for players is the same.

\( h_t = q^0, q^1, q^2, \ldots, q^{t-1}, q^t \) = sequence of game, a history of steps at time \( t \)

\( H_t = \) set of all possible histories of length \( t \)

\[ \Pi_t = H_t A_i = \) set of deterministic strategies

A behavioral strategy \( S_i(h_t, a_{ij}) \) returns the probability of playing action \( a_{ij} \) for history \( h_t \)

A Markov strategy \( S_i \) is a Markovian strategy in which \( S_i(h_t, a_{ij}) = \)

\[ S_i(h'_t, a_{ij}) \]

if \( q_t = h_t \), where these are final states of \( h_t \) and \( h'_t \)

A stationary strategy \( S_i \) is a Markov strategy in which

\[ S_i(h'_t, a_{ij}) = S_i(h''_t, a_{ij}) \] if \( h'_t = h''_t \) with these as final states of \( h'_t \) and \( h''_t \)

Theorem: Every n-player, general-sum, discounted-reward stochastic game has a Markov Perfect Equilibrium (MPE).

An MPE consists of only Markov strategies and is a Nash Equil.

regardless of the prior state.

Hexmoor
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Consider a 2-player zero-sum game with 2 states: $W_1$ and $W_2$.

**Payoff matrix for $W_1$:**

\[
\begin{pmatrix}
10 & -1 \\
-1 & 10
\end{pmatrix}
\]

**For $W_2$:**

\[
\begin{pmatrix}
0 & 6 \\
3 & 0
\end{pmatrix}
\]

(Immediate Payoffs)

**Transition Probabilities for $W_1$:**

\[
\begin{pmatrix}
\frac{1}{2}, \frac{1}{2} \\
\frac{1}{2}, \frac{1}{2}
\end{pmatrix}
\]

† Prob of going to $W_2, W_2$ after column 1 from row player

For $W_2$:

\[
\begin{pmatrix}
(0, 1) & (0, 1) \\
(0, 1) & (0, 1)
\end{pmatrix}
\]

Let $u =$ Value of staying at state $W=1$

\[
v = \frac{1}{2} \cdot 10 + \frac{1}{2} \cdot (1 \cdot n + 0 \cdot u) = \frac{1}{2} \cdot (-1) + \frac{1}{2} \left( \frac{1}{2} \cdot n + \frac{1}{2} \cdot u \right)
\]

\[
u = \frac{1}{2} \cdot (-1) + \frac{1}{2} \left( \frac{1}{2} \cdot n + \frac{1}{2} \cdot u \right)
\]

\[
u = \frac{1}{2} \cdot n + \frac{1}{2} \cdot n \forall (0, 6) = \frac{1}{2} \cdot n + \frac{1}{2} \cdot \frac{3}{8} = 2
\]

\[
u = \frac{1}{2} \cdot n + \frac{1}{2} \cdot n \forall (3, 0) = \frac{1}{2} \cdot n + \frac{1}{2} \cdot n = 4
\]

Row should play \(\left(\frac{1}{2}, \frac{1}{2}\right)\) at $W=1$ and \(\left(\frac{1}{2}, \frac{3}{8}\right)\) at $W=2$.

Column: \(\left(\frac{2}{3}, \frac{1}{3}\right)\)