Mechanism Design (M.D)

Strategic Social Choice, Implementation Theory, Inverse Game Theory

M.D. produces games that have intended equilibria for players.
Design determines player choices & consequences. No coercion is allowed. Designer's outcome depends on player's choices.(Dutta, 1994)

E.G. 1 - The Commons Problem

A. Government auctions off the rights to the common resource to highest bidder each year; restricted privatization.

B. The government auctions off the rights to extract from the common once & for all...

i.e., Privatization

E.G. 2 - Selling the Da Vinci Diaries

Assume Sotheby's is selling; Bill Gates is the most prominent buyer.

Gates' worth, i.e., private value? Either an aficionado or just a fan.

Sotheby options:

1. Post a high aficionado price & risk low sale.

2. Post two prices: at high price, guarantee sale
   At a low price, item might be withdrawn from sale.

Optimal mechanism is when expected sale price is maximum.

E.G. 3 - Public Auction - Sotheby's Optimo:

1. English auction - 1st Price Open Cry

2. 2nd Price (Vickrey)

Analysis of E.G. 2 with Gates only.

Let θ be utility if Gates is an aficionado

Let \( \mu \) = "fan (\( \theta > \mu > 0 \))

\( p = \) Price Paid

\( \theta - \mu = \) Surplus as an aficionado

\( \epsilon = \mu = \) \"a fan\"
\( p = \text{Sotheby's guess that Gatos is an aficionado} \)

Sotheby's expected value = \( p \theta + (1 - p) \mu \)

Special numbers: \( \theta = \$40 \text{M}, \mu = \$10 \text{M}, p = \frac{1}{2} \)

S's expected revenue = \( p \theta + (1 - p) \mu = \$25 \text{M} \).

If Sotheby does not know Gato's interest level, options:

1. Ask the buyer and choose accordingly; \( p(\theta) \) = Price if aficionado
   \[ p(\mu) = \text{'''' for} \]
   \( \text{If } p(\theta) > p(\mu), \text{ buyer will not report } \text{as an aficionado} \)
   \( \text{Then price will be set at } p(\mu). \)
   \( \text{If } p(\theta) = p(\mu), \text{ buyer will be truthful}. \)

2. Set a flat rate of \( p \).

3. Guarantee sale at a high price, pay \( \frac{\theta}{2} \)
   At low price, there could be 50% chance of withdrawal.

S's expected value = \( p \frac{\theta}{2} + (1 - p) \frac{\mu}{2} \); for SN, EV = $12.5 \text{M}

4. Guaranteed sale at a high price of \( p \)

Prob of withdrawal is \( \frac{\theta}{\mu} \) to sell as a low price of \( q \). \( \theta = \text{Prob of keeping an aficionado would like } \theta \text{ if } \theta - p \geq q \cdot (\theta - q) \)

\[ \text{i.e. } \theta \geq \frac{p - q}{1 - q} \]

\( \text{fan would like it if } q (\mu - p) \geq \mu - p \)

\[ \text{i.e. } \mu \leq \frac{p - q}{1 - q} \]

(22.1 Durre)

Incentive Compatibility Constraint.

To prevent coercion \( p \geq p, \mu \geq p \)

Expected value = \( p \theta + (1 - p) q \)

\( q \)
Revelation Principle

A mechanism is a game (or a set of rules) that specifies the
strategies that the player can choose from and the outcome for each choice.
(e.g., a teacher/parent)

S = a strategy; e.g., accept high priced sure offer or low price uncertain offer
O = an outcome; e.g., buy pay $2 & get the item.
π(S, O, θ) = payoff a player of type θ playing S for outcome O
This is the designer control.

Let (S*, O*, θ) for θ player, (S', O') for μ player.
This assignment is incentive compatible if
π(S*, O*, θ) ≥ π(S, O, θ) ∀ S, O
π(S', O', μ) ≥ π(S, O, μ) ∀ S, O
i.e. θ prefers S*, O* & μ prefers S', O'.
Each player has an outside option, non MD option, To.

MD is free of coercion when
π(S*, O*, θ) ≥ To \( \bigcup \) Individual rationality constraints.
π(S', O', μ) ≥ To

Designer of MD is also known as the Principal.

Direct-revelation Mechanism: A mechanism where the strategy set of a
player is simply to report of its type. Players see to their interest to
tell the truth.

Proposition: For any mechanism that is incentive compatible, individually
rational assignment, there is a direct-revelation mechanism in which
truth telling is incentive compatible. The Principal can restrict
attention to direct-revelation mechanisms & truth telling assignment
within those mechanisms.