Transmission Control Protocol

TCP protocol game in normal form

When Internet congestion occurs, backoff from heavy traffic (c)

Keep on your colleague/peer

Low

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-1,-1</td>
<td>-4,0</td>
</tr>
<tr>
<td>D</td>
<td>0,-4</td>
<td>-3,-3</td>
</tr>
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</table>

i.e., Prisoner's Dilemma

If randomness in the environment, games will be Bayesian.

If time is involved, extensive games.

Definition: An n-person game is a tuple \((N, A, u)\)

where \(n\) players set of action profile \(a \in (a_1, \ldots, a_n)\)

\(u = (u_1, \ldots, u_n)\) payoff/real-valued utility

Prisoner's Dilemma: Outcomes = A

Generalized TCP:

<table>
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<tr>
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<th>C</th>
<th>D</th>
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<tbody>
<tr>
<td>C</td>
<td>a,a</td>
<td>b,c</td>
</tr>
<tr>
<td>D</td>
<td>c,b</td>
<td>d,d</td>
</tr>
</tbody>
</table>

If \(c > a > d > b\)

Sometimes \(a > \frac{b+c}{2}\)

Common payoff game: \(\forall\) action profile \(a\), \(\forall\) agents \(i, j\):

\(U_i(a) = U_j(a)\)

= Pure Coordination game

= Team game

\(e.g., S_1 \begin{array}{|c|c|}
\hline
S_1 & S_2 \\
\hline
1,1 & 0,0 \\
\hline
\end{array}
\)

Pure Coordination

Pure Competition

Hexmoor

2010
Zero-Sum Game: \( \forall i \in \text{agents} \land z \in \mathbb{Z}, u_i(a) + u_z(a) = 0 \)

Pure Competition

Constant-Sum Game: \( \forall i \in \text{agents} \land z \in \mathbb{Z}, u_i(a) + u_z(a) = c \)

e.g., Matching Pennies:

\[
\begin{array}{c|cc}
 & H & T \\
\hline
H & 1, -1 & \text{Pure Competition} \\
T & -1, 1 & \\
\end{array}
\]

e.g., Rock, Paper, Scissors:

\[
\begin{array}{c|cccc}
 & R & P & S \\
\hline
R & 0, 0 & -1, 1 & \text{No Play} \\
P & -1, 1 & 0, 0 & -1, 1 \\
S & -1, 1 & 0, 0 & 0, 0 \\
\end{array}
\]

e.g., Battle of Sexes:

\[
\begin{array}{c|cc}
 & LW & WL \\
\hline
\text{LW} & 2, 1 & 0, 0 \\
\text{WL} & 1, 2 & \\
\end{array}
\]

LW = Lethal Weapon
WL = Wonderful Love

Pure Strategy Profile: Select specific action for each player.

Mixed Strategy Profile: Randomly choose actions for each player.

Let \( \Pi(x) \) = probability distribution over actions \( x \).

\[ S_i = \Pi(A_i) \]

Mixed Profile = \( S_1, S_2, \ldots, S_n \)

\( S_i(a) \) = Probability of choosing action \( a \) for mixed strategy \( S_i \)

Support of a mixed strategy: A subset of actions that have a positive probability by the mixed strategy.

Expected utility for a mixed strategy

\[
U_i(S) = \sum_{a \in A} u_i(a) \prod_{j \neq i} S_j(a_j)
\]

Hermoor
2010
For a single agent, optimal strategy is one that maximizes its expected payoff.

Pareto-dominance: A strategy profile \( S \) pareto dominates \( S' \) if \( \forall i \in N, u_i(s) \geq u_i(s') \) and \( \exists j \in N \) for which \( u_j(s) > u_j(s') \).

1. Some players are made better off without making any other player worse off.

Pareto-optimal: Strategy \( S \) is pareto optimal, or strictly pareto efficient if \( \forall S' \in S \) that pareto dominates \( S \).

P.6

1. A game, \( \exists \) at least one pareto optimal strategy.
2. Some games will have multiple optima.
3. In zero-sum games, all profiles are strictly pareto efficient.
4. In common-payoff games, all pareto optimal strategies have the same payoff.

Define \( S_{-i} = (s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n) \): A profile that leaves out agent \( i \)’s strategy.

Best response: Player \( i \)’s best response to profile \( S_{-i} \) is a mixed strategy \( S_i^* \) such that

\[
u_i(S_i^*, S_{-i}) \geq u_i(S_i, S_{-i}) \quad \text{for all} \quad S_i \in S.
\]

Nash Equilibrium: A profile \( S = (s_1, \ldots, s_n) \) is a Nash equilibrium if, for all agents \( i \), \( s_i \) is a best response to \( S_{-i} \).

Nash Equilibrium is stable, since no agent would want to change his strategy if he knew what others were following.
Strict Nash: A profile $S = (s_1, \ldots, s_n)$ is strict $N$ if $\forall i \forall s' \neq s, u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$.

Weak Nash: A profile $S$ is weak $N$ if $\forall i, \forall s' \neq s, u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i})$ and $S$ is not a strict $N$.

Husband

LW  WL
wife  2,1  0,0
WL   0,0

Computing mixed strategy equilibrium

Assume husband plays $LW$ with probability $p$, $WL$ with probability $1-p$.
If wife mixes her strategies, then she must be indifferent between them.

$u_{wife}(LW) = u_{wife}(WL)$

$2 \cdot p + 0 \cdot (1-p) = 0 \cdot p + 1 \cdot (1-p)$

$\Rightarrow p = \frac{1}{3}$

To make the husband indifferent to his choice, wife must choose LW with prob $\frac{2}{3}$ & WL with prob $\frac{1}{3}$.

All mixed strategy E are weak Nash E.

Expected payoff of both is $\frac{2}{3}$.

Pure strategy Nash dominates mixed strategy.

E.g. matching Pennies does not have a pure Nash E.

Hexmoor
2010
Theorem: Every game with a finite number of players (Nash, 1951) and action profiles has at least one Nash Equilibrium.

Definition: The maxmin strategy for player $i$ is

$$\arg \max_{s_i} \min_{s_{-i}} u_i(s_i, s_{-i}),$$

and the maxmin value for player $i$ is

$$\max_{s_i} \min_{s_{-i}} u_i(s_i, s_{-i}).$$

Theorem (Von Neumann, 1928): In any finite, two player zero-sum game, in any N.E. each player receives a payoff that is equal to both his maxmin value and his minimax value.

Regret: An agent $i$’s regret for playing an action $a_i$ if the other agents adopt action profile $a_{-i}$ is defined as

$$\max_{a_{-i} \in A_{-i}} \left( \max_{a_i' \in A_i} u_i(a_i', a_{-i}) - u_i(a_i, a_{-i}) \right)$$

The best response is what you could have had.

$$\max \text{ regret}$$

$$\max_{a_{-i} \in A_{-i}} \left( \max_{a_i' \in A_i} u_i(a_i', a_{-i}) - u_i(a_i, a_{-i}) \right)$$

$$\text{best possible response}$$

$$\frac{u_i(a_i, a_{-i})}{\text{actual}}$$

$$\min \text{imax regret}$$

$$\arg \max \min_{a_i \in A_i} \left( \max_{a_i' \in A_i} u_i(a_i', a_{-i}) - u_i(a_i, a_{-i}) \right)$$