1. Given that $A \cdot B = 0$ and $A + B = 1$, use algebraic manipulation to prove that:

$$ (A + C) \cdot (A + B) \cdot (B + C) = (B \cdot C) $$

2. Simplify the following Boolean expressions to the indicated number of literals:

   (a) $\overline{AC} + ABC + BC$

   (b) $(A + B)(\overline{A} + \overline{B})$

   (c) $ABC + \overline{AC}$

   (d) $BC + B(AD + \overline{CD})$

   (e) $(B + \overline{C} + \overline{B}C)(BC + AB + AC)$

3. Reduce the following Boolean expressions to the indicated number of literals:

   (a) $\overline{XY} + XYZ + \overline{XY}$ to three literals

   (b) $X + Y(Z + \overline{X} + \overline{Z})$ to two literals

   (c) $(AB + \overline{AB})(\overline{CD} + CD) + \overline{AC}$ to four literals

4. Find the complement of the following expressions:

   (a) $\overline{AB} + \overline{AB}$

   (b) $(\overline{V}W + X)Y + \overline{Z}$

   (c) $WX(\overline{Y}Z + Y\overline{Z}) + \overline{W}X(\overline{Y} + Z)(Y + \overline{Z})$
5. Obtain the truth table of the following functions, and express each function in sum-of-minterms and product-of-maxterms form:

(a) \((XY + Z)(Y + XZ)\)

(b) \((\overline{A} + B)(\overline{B} + C)\)

(c) \(WX\overline{Y} + WX\overline{Z} + WXZ + Y\overline{Z}\)

6. For the Boolean functions E and F, as given in the following truth table:

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(a) List the minterms and maxterms of each function

(b) List the minterms of \(E + F\) and \(E \cdot F\)

(c) Express E and F in sum-of-minterms algebraic form

(d) Simplify E and F to expressions with a minimum of literals