**CS 330 Intro to the Design and Analysis of Algorithms**

Homework 4

**(20 pts)**

1. Let G be a graph where every edge has a distinct weight. Show

that:

a. The edge with the smallest edge weight is in the minimum spanning tree.

b. There is only one, unique minimum spanning tree in this case. [3 points]

2. In the weighted graph from the figure below, find the sequence of edge weights

selected when:

A picture containing photo, building, boat, wire

Description automatically generated

1. Kruskal’s algorithm is run.
2. Prim’s algorithm is run. [4 points]

3. Explain the differences between P, NP, and NP-Complete problems. Find a topological ordering for the following directed acyclic graph: [4 points]

A clock in the middle of a watch

Description automatically generated

4. A unit-time task is a job, such as a program to be run on a computer, that requires exactly one unit of time to complete. Given a finite set S of unit-time tasks, a schedule for S is a permutation of S specifying the order in which to perform these tasks. The first task in the schedule begins at time 0 and finishes at time 1, the second task begins at time 1 and finishes at time 2, and so on.

The problem of scheduling unit-time tasks with deadlines and penalties for a single processor has the following inputs:

* a set S = {a1, a2,….,an} of n unit-time tasks;
* a set of n integer deadlines d1, d2,…., dn, such that each di satisfies 1<=di<=n

and task ai is supposed to finish by time di; and

* a set of n nonnegative weights or penalties w1, w2,…., wn, such that we incur

a penalty of wi if task ai is not finished by time di, and we incur no penalty if

a task finishes by its deadline.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ai | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| di | 4 | 2 | 4 | 3 | 1 | 4 | 6 |
| wi | 70 | 60 | 50 | 40 | 30 | 20 | 10 |

Consider the above table. Propose a greedy algorithm and use that to find a schedule for S that minimizes the total penalty incurred for missed deadlines. [5 points]

5. In the weighted graph from the figure below, find the sequence of edge weights

selected when:

A picture containing photo, building, boat, wire

Description automatically generated

Dijkstra’s algorithm is run (r is the start vertex). [4 points]