CS 330 Intro to the Design and Analysis of Algorithms

Homework 4

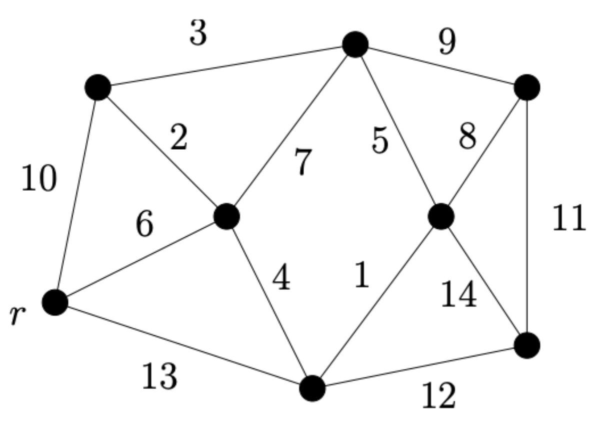
(20 pts)

1. Let G be a graph where every edge has a distinct weight. Show that:

   a. The edge with the smallest edge weight is in the minimum spanning tree.

   b. There is only one, unique minimum spanning tree in this case. [3 points]

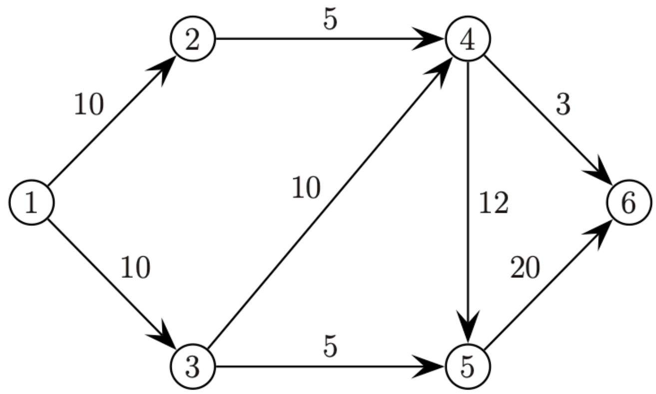
2. In the weighted graph from the figure below, find the sequence of edge weights selected when:



   a. Kruskal's algorithm is run.

   b. Prim's algorithm is run. [4 points]

3. Explain the differences between P, NP, and NP-Complete problems. Find a topological ordering for the following directed acyclic graph: [4 points]



4. A unit-time task is a job, such as a program to be run on a computer, that requires exactly one unit of time to complete. Given a finite set S of unit-time tasks, a schedule for S is a permutation of S specifying the order in which to perform these tasks. The first task in the schedule begins at time 0 and finishes at time 1, the second task begins at time 1 and finishes at time 2, and so on.

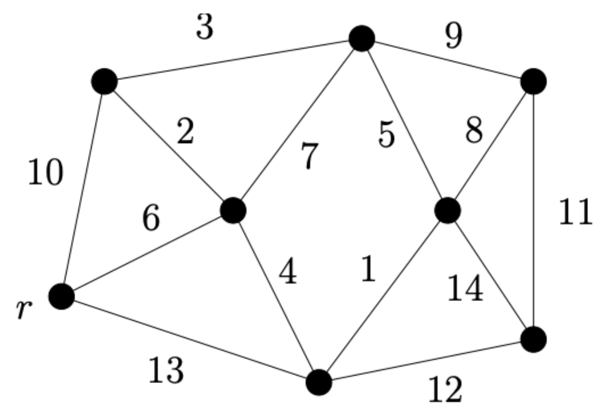
The problem of scheduling unit-time tasks with deadlines and penalties for a single processor has the following inputs:

- a set $S = \{a_1, a_2, \ldots, a_n\}$ of n unit-time tasks;
- a set of n integer deadlines $d_1, d_2, \ldots, d_n$, such that each d_i satisfies $1 <= d_i <= n$ and task a_i is supposed to finish by time d_i; and
- a set of n nonnegative weights or penalties $w_1, w_2, \ldots, w_n$, such that we incur a penalty of w_i if task a_i is not finished by time d_i, and we incur no penalty if a task finishes by its deadline.

a_i	1	2	3	4	5	6	7
d_i	4	2	4	3	1	4	6
w_i	70	60	50	40	30	20	10

Consider the above table. Propose a greedy algorithm and use that to find a schedule for S that minimizes the total penalty incurred for missed deadlines. [5 points]

5. In the weighted graph from the figure below, find the sequence of edge weights selected when:



Dijkstra's algorithm is run (r is the start vertex). [4 points]