

# Joint Variable Width Spectrum Allocation and Link Scheduling for Wireless Mesh Networks

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**Abstract**—In wireless mesh networks with frequency-agile radios, an approach of dynamically combining consecutive channels has recently been proposed. However, the available channel widths are limited in the approach. In order to further improve the fairness or the throughput under given fairness, we propose a joint variable width spectrum allocation and link scheduling optimization algorithm. Our algorithm is composed of time division multiple access for no interface conflict and frequency division multiple access for no signal interference. In the first phase, we use as few time slots as possible to assign at least one time slots to each radio link with Max-Min fairness. In the second phase, our design jointly allocates the lengths of time slots as well as the spectral widths and center frequencies of radio links in each time slot. Numerical results indicate that compared to the existing algorithm, our algorithm significantly increases the fairness or the throughput under given fairness.

**Keywords**—spectrum allocation; link scheduling; variable channel width; wireless mesh networks

## I. INTRODUCTION

Frequency-agile radio technologies enable more flexible spectrum access, such as dynamic reconfiguration of the center frequencies and spectral widths of communication channels. The combination of variable channel widths and multiple center frequencies offers rich possibilities for improving wireless network performance [1].

Recently, researchers begin to study the allocation of spectral bands of uneven and variable widths. Reference [2] presents a joint channel assignment, link scheduling and routing optimization algorithm for multi-radio wireless mesh networks. The centralized algorithm dynamically combines several consecutive channels into one so that a network interface can use the channel with a larger range of frequencies, and thus improves the channel capacity. Nevertheless, the widths of combined channels are only chosen from several values. Moreover, with the number of channels increasing, the running time of the algorithm mounts up rapidly.

Some research takes into account the case that the spectral width assigned to a radio link can be continuously variable within a predefined range. In [3], Hou et al. formulate spectrum sharing and unequal frequency band division. However, their centralized algorithm assumes that a radio can send packets over non-contiguous frequency bands and receive packets from different radios on the same sub-band. Such a strong assumption is prohibitive in practice. Reference [4] designs a

joint transport, routing and spectrum sharing optimization algorithm, which distributedly allocates both the center frequencies and spectral widths of communication channels. Nonetheless, there is no radio constraint in the formulation in [4]. According to the algorithm, the number of simultaneously scheduled links associated with a router is likely to be larger than the number of the radios in the router. In other words, the interface conflict among these links may occur.

In this paper, we address a joint variable width spectrum allocation and link scheduling problem for fairness optimization or throughput optimization under given fairness in wireless mesh networks, which is NP-hard in general. We propose a centralized near-optimal algorithm, whose time complexity is  $O(m^{3.5})$ , where  $m$  is the number of radio links. Although our algorithm is a centralized algorithm, this does not prohibit it from being implemented in the real world for practical applications (see a detailed explanation in Section VII of [3]). Compared to the related literature, the main contributions of our work are as follows:

- Our algorithm can raise the fairness among flows in wireless mesh networks by 46 to 76%, or increase the aggregated throughput under given fairness by 90 to 213%.
- Our algorithm can efficiently limit the channel switching cost. The total number of time slots in a period does not exceed  $\Delta + 2$ , where  $\Delta$  is the maximum degree of a directed network graph.
- Our algorithm, which takes the radio constraint into account, can be suitable for wireless mesh networks with any number of radios per node. Additionally, it does not rely on any interference model.

The rest of this paper is organized as follows. Section II describes our system model to depict a multi-radio wireless mesh network. In Section III, we decompose the joint variable width spectrum allocation and link scheduling problem into two subproblems: time division multiple access (TDMA) and frequency division multiple access (FDMA). Then we propose a mathematical formulation for each subproblem and derive their approximately optimal solutions. In Section IV, we give numerical results, which demonstrate a great performance improvement of our algorithm in comparison with the algorithm in [2]. Some conclusions are provided in Section V.

## II. A SYSTEM MODEL

Suppose that there is a set  $V$  of routers and a set  $K (\subset V)$  of gateways in a wireless mesh network. Among the routers, there

is a set  $L$  of unidirectional logical links. In this paper, we use routers and nodes interchangeably. A multi-radio wireless mesh network is represented by a directed network graph  $G(R, E)$ , where each vertex  $r \in R$  corresponds to a radio of a router in the network, and each directed edge  $e \in E$  corresponds to a physical link from one radio to another. Note that there are no edges between two radios in the same router. For a logical link  $l = (u, v)$  and an edge  $e = (p, q)$ ,  $e \in l$  means that the radio  $p$  is an interface of router  $u$  and the radio  $q$  is an interface of router  $v$ .

We need to distinguish two concepts: interface conflict and signal interference. The former denotes that two physical links share the same radio. The latter indicates that two physical links do not share the same radio but they cannot successfully transmit data over two overlapping frequency bands at the same time. We consider the radio constraint that no active physical links simultaneously share the same radio. In order to avoid interface conflict, time needs to be divided into slots, and every two physical links sharing the same radio must be allocated to different time slots. Note that some physical links interfering with each other is likely to be assigned to the same time slot. We say that  $(\hat{e}, t) \in I(e, t)$  if there is signal interference between  $e$  and  $\hat{e}$  in time slot  $t$ . Associated with a network graph  $G(R, E)$ , we can define an interference graph  $G_t^I = (E_t, I_t)$  for each time slot  $t$ , where  $E_t \subset E$  and there is an undirected edge between vertices  $e$  and  $\hat{e}$  in  $G_t^I$  if  $(\hat{e}, t) \in I(e, t)$ .

### III. PROBLEM FORMULATION FOR JOINT SPECTRUM ALLOCATION AND LINK SCHEDULING

In this section, we describe a mathematical model to formulate a joint variable width spectrum allocation and link scheduling optimization problem, and present our algorithm for the problem. The goal of our algorithm is to maximize the fairness or the throughput under given fairness among all uploading (or downloading) flows in a wireless mesh network.

The basic idea behind our algorithm is decomposing the joint optimization problem into the following two subproblems. The first subproblem is TDMA link scheduling for no interface conflict in each time slot. The objective of the subproblem is to minimize the total number of time slots in a scheduled period, under the condition that each physical link is scheduled in at least one time slot and the radio constraint is satisfied in each time slot. Additionally, we assign as many time slots as possible to each physical link with Max-Min fairness. The second subproblem is FDMA spectrum allocation for no signal interference in each time slot, based on the time slot assignment results of the first subproblem. The objective of the second subproblem is to jointly allocate the spectral widths and center frequencies of physical links in each time slot as well as the lengths of time slots to achieve the maximum fairness or the maximum throughput under given fairness.

#### A. TDMA Link Scheduling

In this subsection, we describe a mathematical model to formulate the TDMA link scheduling subproblem. The solution procedure of this subproblem (see Algorithm 1 for more details) is divided into two steps. The first step is to use a minimum number of time slots to assign each physical link a time slot under the radio constraint. Given the total number of time slots, the second step is to maximize the number of time slots assigned to each physical link with Max-Min fairness.

Algorithm 1. TDMA Link Scheduling  
Input: A directed network graph  $G(R, E)$  of  $n$  vertices and  $m$  edges  
Output: A link scheduling policy without interface conflict,  $S$  (a set of pairs of an edge and a time slot)  
Global variables:  $weight[m]$ , an array of  $m$  elements containing the weight of each edge

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1:  $S \leftarrow \emptyset$ 
2:  $T \leftarrow \text{Edge\_Coloring}(G)$ 
3: for  $e \leftarrow e_1$  to  $e_m$  do //  $e_1, \dots, e_m \in E$ 
4:    $weight[e] \leftarrow T - 1$ 
5: for  $t \leftarrow 1$  to  $T$  do
6:   for  $r \leftarrow r_1$  to  $r_n$  do //  $r_1, \dots, r_n \in R$ 
7:      $match[r] = -1$  // Initialization
8:   for  $e \leftarrow e_1$  to  $e_m$ 
9:     if  $color[e] = t$  then
10:       $match[src(e)] = dst(e)$  //  $src(e)$  and  $dst(e)$  are the
initial and terminal vertices of  $e$ , respectively.
11:       $match[dst(e)] = 0$ 
12:   Maximal_Weight_Scheduling( $G, match$ )
13:   for  $r \leftarrow r_1$  to  $r_n$  do
14:     if  $match[r] > 0$  then
15:        $S \leftarrow S \cup \{(r, match[r], t)\}$ 

```

Function 1-1. Edge\_Coloring()  
Input:  $G(R, E)$ , a directed network graph of  $m$  edges  
Output:  $color[m]$ , an array of  $m$  elements containing the colors of all directed edges  
Return:  $T$ , the total number of used colors

Function 1-2. Maximal\_Weight\_Scheduling()  
Input:  $G(R, E)$ ;  $match[n]$ , an array of  $n$  elements recording the initial edges scheduled in current time slot  
Output:  $match[n]$ , the edges scheduled in current time slot

```

1: Sort the directed edges in  $G(R, E)$  from the maximum to
minimum weight. Let  $(\hat{e}_1, \hat{e}_2, \dots, \hat{e}_m)$  be the sorted list of edges.
2: for  $e \leftarrow \hat{e}_1$  to  $\hat{e}_m$  do
3:   if  $match[src(e)] < 0$  and  $match[dst(e)] < 0$  then
4:      $match[src(e)] = dst(e)$ 
5:      $match[dst(e)] = 0$ 
6:    $weight[e] \leftarrow weight[e] - 1$ 

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#### 1) Minimizing the Number of Time Slots in a Period

In order to reduce the channel switching cost, we minimize the total number of time slots in a period. The problem of assigning a time slot to each physical link using a minimum number of time slots can be viewed as an edge coloring problem in the directed network graph  $G(R, E)$ . A time slot is mapped to a color, and a physical link is a directed edge in the network graph. The edge coloring problem is known to be NP-complete [5]. The network graph  $G$  is a multi-graph with the maximum degree  $\Delta$  and the maximum edge multiplicity  $\mu = 2$ . According to Vizing's theorem [6], the edge chromatic number  $\chi'(G)$  of  $G$  is at most  $\Delta + \mu$ . Therefore,  $\Delta \leq \chi'(G) \leq \Delta + 2$ .

Misra and Gries [7] provided a constructive proof of Vizing's theorem. Given an arbitrary simple graph, the constructive algorithm provides a valid edge coloring using at most  $\delta + 1$  colors, where  $\delta$  is the maximum degree of the graph. For a detailed description and a proof of correctness, we refer the reader to [7]. For our network graph, we convert two opposite directed edges between two vertices to an undirected edge. The new graph is a simple graph whose maximum degree is  $\Delta / 2$ , so we can color its edges using at most  $\Delta / 2 + 1$  colors, based on the constructive proof. In our directed network graph, every two opposite directed edges individually have the  $2c-1$ th and  $2c$ th color, where the  $c$ th color is the color of the

corresponding undirected edge. Thus, we can obtain a feasible time-slot allocation that uses at most  $\Delta + 2$  time slots.

## 2) Maximal Weight Scheduling

Based on the result of the first step, the second step of the TDMA link scheduling subproblem is to assign as many time slots as possible to each physical link with Max-Min fairness. The purpose of this step is to make each physical link have more opportunities to be active.

Our principle is that a physical link allocated less time slots has a higher priority. We assign a weight to each edge, where the weight value equals the total number of time slots minus the number of time slots assigned to the edge and is dynamically updated. For each time slot, we rank all edges according to their current weights, and find a maximal weight match in the network graph, based on the scheduled edges in the first step. In each time slot, all the edges in the maximal weight match are scheduled.

The time complexity of the edge coloring algorithm is  $O(m^2)$ , where  $m$  is the number of directed edges in the network graph. The time complexity of the maximal weight scheduling algorithm is  $O(n \log n)$ , where  $n$  is the number of vertices in the network graph. Therefore, Algorithm 1 has the computational complexity of  $O(m^2)$ .

## B. Spectrum Allocation and Time-Slot Length Adjustment

In this subsection, we present a 0-1 mixed integer linear programming (MILP) formulation of the joint FDMA spectrum allocation and time-slot length adjustment subproblem, and develop a near-optimal algorithm to solve the formulation for maximizing the fairness or the throughput under given fairness.

### 1) Modeling

We mathematically formulate the joint optimization problem by taking into account flow conservation constraints, link capacity constraints, time-slot length constraints and spectrum constraints.

*a) Flow Conservation Constraints:* We are given a flow demand  $S_v$  from each source node  $v$ .  $r_v$  is the achieved flow from the source node  $v$ . The achieved fairness  $\lambda$  is defined as the minimum ratio of the achieved flow to the demanded load among all nodes. We denote by  $g_{in}(v)$  and  $g_{out}(v)$  the total aggregated incoming and outgoing traffic of  $v$ , respectively.  $h_l$  represents the total scheduled traffic over logical link  $l$ .  $L(v)$  and  $L^+(v)$  are the sets of unidirectional logical links that start and end at node  $v$ , respectively. A feasible flow allocation must follow the flow conservation constraints below [2].

$$\begin{cases} r_v \geq \lambda S_v, \forall v \\ g_{in}(v) + r_v = g_{out}(v), \forall v \\ g_{in}(v) = \sum_{l \in L^-(v)} h_l, \forall v \\ g_{out}(v) = \sum_{l \in L^+(v)} h_l, \forall v \notin K \end{cases} \quad (1)$$

*b) Link Capacity Constraints:* We denote by  $c_l$  the capacity of logical link  $l$ .  $c_{e,t}$  and  $b_{e,t}$  represent the capacity and spectral width of physical link  $e$  in time slot  $t$ , respectively.  $SNR_l$  is the signal-to-noise ratio of all the physical links of logical link  $l$ . A period is divided into  $T$  time slots.  $x_t$  indicates the ratio of the length of time slot  $t$  to the length of a period.

$$\begin{cases} h_l \leq c_l, \forall l \\ c_l = \sum_{1 \leq t \leq T} x_t \sum_{e \in l} c_{e,t} = \log(1 + SNR_l) \sum_{1 \leq t \leq T} \sum_{e \in l} x_t b_{e,t}, \forall l \end{cases} \quad (2)$$

The first constraint in (2) ensures that the traffic over any link cannot exceed its capacity. The second equality in (2) expresses the link capacity according to Shannon's theorem.

*c) Time-Slot Length Constraints:* The length of a period is equal to the sum of the lengths of all time slots.

$$\begin{cases} \sum_{1 \leq t \leq T} x_t = 1 \\ x_t \geq 0, \forall t \end{cases} \quad (3)$$

*d) Spectrum Constraints:* The center frequency of physical link  $e$  in time slot  $t$  is denoted by  $f_{e,t}$ . Assume that the available spectrum band is from 0 to  $B$ . A spectrum allocation is feasible if and only if the following inequalities hold.

$$\begin{cases} 2|f_{e,t} - f_{\hat{e},t}| \geq b_{e,t} + b_{\hat{e},t}, \forall (e,t), \forall (\hat{e},t) \in I(e,t) \\ f_{e,t} + b_{e,t}/2 \leq B, \forall e, \forall t \\ f_{e,t} - b_{e,t}/2 \geq 0, \forall e, \forall t \\ b_{e,t} \geq 0, \forall e, \forall t \end{cases} \quad (4)$$

The first constraint in (4) is the interference constraint, and reflects that if there is signal interference between any two physical links in any time slot, they must operate in non-overlapping frequency sub-bands. The second and third constraints limit frequency sub-bands of all physical links to the range of the available frequency band.

*e) Fairness or Throughput Maximization:* Our optimization objective is to maximize the fairness among all the flows or the total throughput under given fairness by jointly adjusting the time-slot lengths, the spectral widths and center frequencies. The first design goal can be summarized in the following constrained optimization problem.

$$\begin{aligned} \Lambda &= \max \lambda \\ \text{s.t. } &(1), (2), (3) \text{ and } (4) \text{ hold.} \end{aligned} \quad (5)$$

Given achieved fairness  $\lambda$  ( $0 \leq \lambda \leq \Lambda$ ), the second design goal can be formalized as

$$\begin{aligned} \max \sum_{v \in (V-K)} r_v \\ \text{s.t. } &(1), (2), (3) \text{ and } (4) \text{ hold.} \end{aligned} \quad (6)$$

### 2) An Equivalent 0-1 MILP Formulation

The problem formulation in the above subsection has a non-linear constraint, say the second equality in (2), and the feasible region is the union of some disjoint regions due to the first inequality in (4). For such a non-convex optimization problem, we can only compute its local optimal solution. Hence, in this subsection, we transform the problem into an equivalent 0-1 MILP problem to approximate the global optimal solution.

In order to eliminate the quadratic term in (2), we define real auxiliary variables  $\alpha_{e,t}$  and  $\beta_{e,t}$ .

$$\begin{cases} \alpha_{e,t} = x_t b_{e,t}, \forall e, \forall t \\ \beta_{e,t} = x_t f_{e,t}, \forall e, \forall t \end{cases} \quad (7)$$

We replace  $x_t b_{e,t}$  in (2) with a single variable, say  $\alpha_{e,t}$ , so that the product in the second equality in (2) disappears. We multiply all the constraints in (4) by the corresponding  $x_t$ , and

then let  $\alpha_{e,t}$  and  $\beta_{e,t}$  appear throughout as products instead of  $x_t b_{e,t}$  and  $x_t f_{e,t}$ , respectively. Therewith, (4) becomes (8).

$$\begin{cases} 2|\beta_{e,t} - \beta_{\hat{e},t}| \geq \alpha_{e,t} + \alpha_{\hat{e},t}, \forall(e,t), \forall(\hat{e},t) \in I(e,t) \\ \beta_{e,t} + \alpha_{e,t} / 2 \leq Bx_t, \forall e, \forall t \\ \beta_{e,t} - \alpha_{e,t} / 2 \geq 0, \forall e, \forall t \\ \alpha_{e,t} \geq 0, \forall e, \forall t \end{cases} \quad (8)$$

Owing to the first constraint in (8), the optimization problem is divided into  $2^p$  linear programming (LP) subproblems, where  $p$  is the number of the inequalities in the first constraint in (8). In order to incorporate these subproblems into one problem, we define a series of 0-1 binary auxiliary variables  $Y_{e,\hat{e},t}$ .

$$Y_{e,\hat{e},t} = [1 + \text{sgn}(f_{e,t} - f_{\hat{e},t})] / 2, \forall(e,t), \forall(\hat{e},t) \in I(e,t) \quad (9)$$

where  $\text{sgn}(\cdot)$  is the signum function and returns 1 or -1.

Since  $x_t \geq 0$ , the first constraint in (8) is equivalent to (10).

$$2(\beta_{e,t} - \beta_{\hat{e},t})(2Y_{e,\hat{e},t} - 1) \geq \alpha_{e,t} + \alpha_{\hat{e},t}, \forall(e,t), \forall(\hat{e},t) \in I(e,t) \quad (10)$$

With both sides added by  $4BY_{e,\hat{e},t}$ , (10) becomes

$$4(\beta_{e,t} - \beta_{\hat{e},t} + B)Y_{e,\hat{e},t} - 2(\beta_{e,t} - \beta_{\hat{e},t}) \geq \alpha_{e,t} + \alpha_{\hat{e},t} + 4BY_{e,\hat{e},t}, \quad (11) \\ \forall(e,t), \forall(\hat{e},t) \in I(e,t)$$

The first term on the left side of the above inequality is the product of a bounded non-negative real variable (less than  $2B$ ) and a binary variable. With the linearization technique in Appendix B of [8], the quadratic constraint can be linearized. We replace the product by a real auxiliary variable  $\gamma_{e,\hat{e},t}$ .

$$\gamma_{e,\hat{e},t} = (\beta_{e,t} - \beta_{\hat{e},t} + B)Y_{e,\hat{e},t}, \forall(e,t), \forall(\hat{e},t) \in I(e,t) \quad (12)$$

Then, we substitute the linear inequalities in (13) for (12).

$$\begin{cases} 0 \leq \gamma_{e,\hat{e},t} \leq \beta_{e,t} - \beta_{\hat{e},t} + B, \forall(e,t), \forall(\hat{e},t) \in I(e,t) \\ \beta_{e,t} - \beta_{\hat{e},t} + B - 2B(1 - Y_{e,\hat{e},t}) \leq \gamma_{e,\hat{e},t} \leq 2BY_{e,\hat{e},t}, \forall(e,t), \forall(\hat{e},t) \in I(e,t) \end{cases} \quad (13)$$

Finally, we obtain an equivalent 0-1 MILP formulation.

$$\max \lambda \text{ or } \max \sum_{v \in (V-K)} r_v \quad (14)$$

s.t. (1), (3), (13), (2) with the substitution of  $\alpha_{e,t}$  for  $x_t b_{e,t}$ , the last three constraints in (8), and

$$4\gamma_{e,\hat{e},t} - 2(\beta_{e,t} - \beta_{\hat{e},t}) \geq \alpha_{e,t} + \alpha_{\hat{e},t} + 4BY_{e,\hat{e},t}, \forall(e,t), \forall(\hat{e},t) \in I(e,t)$$

### 3) A Near-optimal Algorithm Based on Vertex Coloring

The optimization problem (14) is a 0-1 MILP problem, which is NP-complete in general. Although existing software can solve small-sized network instances, such as several nodes, the time complexity becomes prohibitively high for large-sized networks. Hence, we have to find a near-optimal solution.

Once the values of all binary integer variables  $Y_{e,\hat{e},t}$  are determined, the original MILP is reduced to an LP, which can be solved in polynomial time. According to (9),  $Y_{e,\hat{e},t}$  indicates which of the center frequencies of  $e$  and  $\hat{e}$  is higher in time slot  $t$ . If we determine the hierarchy of the center frequencies of all physical links in each time slot, the values of all  $Y_{e,\hat{e},t}$  are fixed.

In order to effectively exploit spectrum resources, we hope to minimize the number of hierarchy levels. This subproblem can be modeled as a vertex coloring problem on the interference graph  $G^l_t(E_t, I_t)$ . The number of colors needed is the number of hierarchy levels. The color number of a vertex is its level. We use the recursive largest first (RLF) [9] algorithm to color vertices of  $G^l_t(E_t, I_t)$ . The main idea of RLF algorithm is coloring as many vertices with one color as possible and then moving on to the next color. The RLF algorithm prefers to color the vertex with the largest number of uncolored neighbors.

Accordingly, we propose a near-optimal algorithm to approximately solve the MILP problem. Our approach (see Algorithm 2 for more details) is to fix the values of all the integer variables based on vertex coloring, and consists of three steps. (i) Color the vertices of the interference graph  $G^l_t$  by the RLF algorithm. (ii) For each variable  $Y_{e,\hat{e},t}$ , compare the color numbers of  $e$  and  $\hat{e}$ . If the color number of  $e$  is smaller than that of  $\hat{e}$ , then  $Y_{e,\hat{e},t} = 1$ . Otherwise,  $Y_{e,\hat{e},t} = 0$ . (iii) Find an optimal solution of the LP corresponding to the fixed values of all binary variables  $Y_{e,\hat{e},t}$ .

The time complexity of the vertex coloring algorithm is  $O(n^2m)$ . The time complexity of interior point methods for LP is  $O(m^{3.5})$ . Consequently, Algorithm 2 has the computational complexity of  $O(m^{3.5})$ .

## IV. PERFORMANCE EVALUATION

We evaluate the performance of our joint spectrum allocation and link scheduling (SALS) algorithm in comparison with the joint channel combination and link scheduling (CCLS) algorithm ( $C = 1$  for yielding a feasible interference-free link-channel scheduling) in [2].

The WMN consists of 60 nodes and 8 gateways are randomly chosen from the nodes. The nodes are randomly dispersed in a square area of  $500 \times 500 m^2$ . Five fully connected

### Algorithm 2. FDMA Spectrum Allocation

Input: The interference graph  $G^l_t(E_t, I_t)$  of  $m(t)$  undirected edges

$(i_1, i_2, \dots, i_{m(t)} \in I_t)$  in each time slot,  $1 \leq t \leq T$

Output: The values of all the real variables  $x_t$ ,  $b_{e,t}$  and  $f_{e,t}$

- 1: for  $t \leftarrow 1$  to  $T$  do
- 2: Vertex\_Coloring( $G^l_t(E_t, I_t)$ )
- 3: for  $i \leftarrow i_1$  to  $i_{m(t)}$  do
- 4:  $e$  and  $\hat{e}$  are two vertices of edge  $i$ , and the index of  $e$  is smaller than the index of  $\hat{e}$ .
- 5: if  $color[e] < color[\hat{e}]$  then
- 6:  $Y_{e,\hat{e},t} \leftarrow 1$
- 7: else
- 8:  $Y_{e,\hat{e},t} \leftarrow 0$
- 9: By substituting the fixed values (0 or 1) for the integer variables  $Y_{e,\hat{e},t}$ , the 0-1 MILP problem (5) or (6) becomes a LP problem.
- 10: Solve the LP problem with the interior point method, and obtain the values of all the real variables  $x_t$ ,  $b_{e,t}$  and  $f_{e,t}$ .

Function 2-1. Vertex\_Coloring() // Recursive Largest First

Input:  $G^l_t(E_t, I_t)$

Output:  $color[]$ , an array containing the colors of all vertices

- 1:  $color\_count \leftarrow 0$
- 2: while uncolored vertices exist do
- 3:  $color\_count \leftarrow color\_count + 1$
- 4: Make a list  $D$  of all uncolored vertices.
- 5: while  $D$  is not empty do
- 6: Find the vertex  $e$  in  $D$  with the largest number of uncolored neighbors.
- 7:  $color[e] \leftarrow color\_count$
- 8: Remove vertex  $e$  from  $D$  and also remove vertex  $e$ 's neighbors (since they can't be colored this same color).
- 9: Return  $color\_count$  (the number of colors needed)

topologies are selected as evaluation scenarios. Each point in Fig. 1 and Fig. 2 is the average of numerical results on the five topologies. We assume the maximum transmission distance of 90m and the maximum interference distance of 135m. If two nodes want to launch communications, any other link whose node is within the interference range of the two nodes must keep silent. We adopt the same parameters of link capacity as [2]. We vary the maximum number of radios per node from 1 to 6, while using 1 as the minimum number of radios per node.

We evaluate the fairness of the SALS and CCLS algorithms, and observe the affect of using multiple radios per node on the fairness. Figure 1 reveals that our SALS algorithm provides 46% to 76% higher fairness compared with the CCLS algorithm. By our algorithm, the fairness measures increase with the number of radios until a certain point where they start bobbing up and down.

Given fairness  $\lambda$ , we examine the throughput with the SALS and CCLS algorithms, respectively. Here,  $\lambda$  is set to the maximum fairness achieved by the CCLS algorithm. Figure 2 demonstrates that in comparison with the CCLS algorithm, our SALS algorithm increases the throughput by 90% to 213%, and effectively exploits multi-radio resources.

From Fig. 3, we can compare the SALS and CCLS algorithms in terms of the number of time slots in a period. Since the performance of the CCLS algorithm does not vary when the number of radios changes, Fig. 3(a) only plots the case of one radio per node. Figure 3(a) displays the number of time slots used (from 261 to 407) when the CCLS algorithm reach just 50% of its maximum achieved fairness. Figure 3(b) illustrates the number of time slots used (from 15 to 27) by our SALS algorithm in various scenarios when the maximum fairness of our algorithm is achieved. Obviously, our algorithm uses a far smaller number of time slots and efficiently reduces the frequency switching cost.

### V. CONCLUSIONS

In this paper, we present a joint variable width spectrum allocation and link scheduling optimization algorithm for wireless mesh networks. In our algorithm, the joint optimization problem is decomposed into the two subproblems. One is TDMA link scheduling for no interface conflict between radio links in each time slot. The other is joint FDMA spectrum allocation and time-slot length adjustment for no signal interference in each time slot, based on the results of the first subproblem. Extensive numerical results show that our proposed algorithm dramatically increases the fairness or the aggregated throughput under given fairness among all flows, and sharply reduce the frequency switching cost by limiting the number of time slots in a period. Furthermore, our algorithm does not rely on any interference model and is suitable for the wireless networks with any number of radios.

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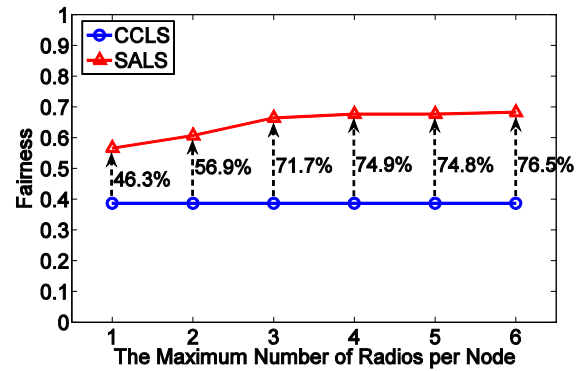


Figure 1. Fairness

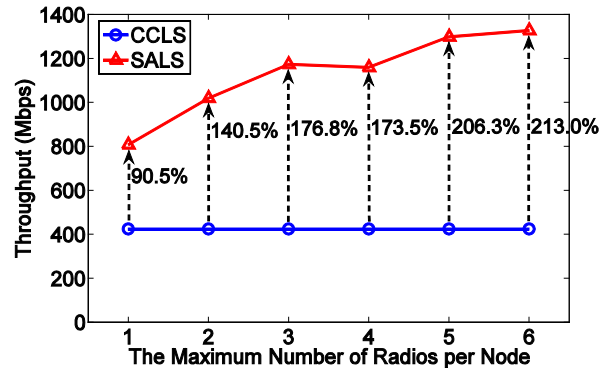
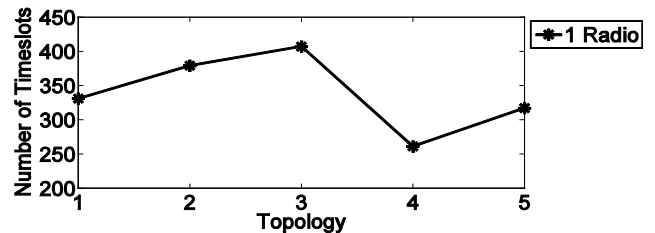
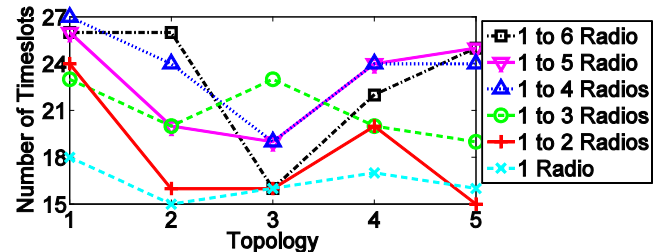


Figure 2. Throughput



(a) Channel Combination and Link Scheduling



(b) Spectrum Allocation and Link Scheduling

Figure 3. The number of time slots

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