

Interference Pair-Based Distributed Spectrum Allocation in Wireless Mesh Networks with Frequency-Agile Radios

Tong Shu^{1,2}, Min Liu¹, Zhongcheng Li¹

¹Institute of Computing Technology, Chinese Academy of Sciences

²Graduate University of Chinese Academy of Sciences
Beijing 100190, P.R. China

Qishi Wu

Department of Computer Science
University of Memphis
Memphis, TN 38152, USA

Abstract—Spectrum allocation algorithms are able to improve the performance of wireless mesh networks by exploiting the frequency agility of modern radios, and several such algorithms have been proposed. However, their interference constraints are at a coarse-grained level, which results in a low spectrum efficiency. To achieve higher spectrum resource utilization, we use interference pairs as a finer granularity to model the interference constraints in wireless mesh networks, and derive a sufficient and necessary condition for interference-free spectrum allocation. Based on a set of rigorous models, we formulate spectrum allocation as an optimization problem and divide it into two subproblems, for which we propose a two-phase interference pair-based distributed spectrum allocation (IPDSA) algorithm. In IPDSA, a negotiation-based frequency hierarchy mechanism heuristically determines the relation between the center frequencies of links in each interference pair; and then a dual decomposition-based spectrum allocation algorithm converges to the optimal allocation of center frequencies and spectral widths of all links. Extensive simulation results show that IPDSA is able to significantly improve spectrum utilization and thus increase network utility and aggregate throughput, thanks to a high accuracy in modeling interference constraints.

Keywords—distributed spectrum allocation; pairwise interference; problem decomposition; frequency agility; wireless mesh networks

I. INTRODUCTION

A frequency-agile radio can dynamically reconfigure its center frequency and spectral width to operate on an arbitrary frequency band over a wide range of spectrum [1]. For instance, a prototype radio capable of transmitting in four channel widths of 5, 10, 20, and 40 MHz has been developed from a commodity 802.11b/g network interface card [2]. An advanced software defined radio (SDR) can simultaneously send and receive packets over non-contiguous frequency bands to and from different radios [3]. The combination of variable spectral widths and multiple center frequencies offers rich possibilities for improving the performance of wireless mesh networks. Unfortunately, even in the fixed channelization framework where the spectral band width is invariable, the spectrum allocation problem is NP-hard in general [4]. The joint allocation of center frequencies and spectral widths further complicates the problem.

Recently, several allocation algorithms with frequency bands of variable widths have been proposed for wireless mesh networks. Reference [5] presents a joint channel assignment, link scheduling and routing optimization algorithm. This algorithm dynamically combines several consecutive channels to improve network throughput, but the spectral width of a combined channel is only chosen from a limited set of values. In [3], Hou *et al.* provide a spectrum sharing algorithm based on unequal frequency band division, in which spectral widths are continuously variable within a predefined range. However, the center frequencies and spectral widths of links on an identical frequency band must be the same. Thus, spectrum access is still not flexible. In addition, the two algorithms in [3] and [5] are both centralized in that the optimization process must be managed by a central entity with global knowledge about the network. In these centralized schemes, any change in the network must be reported to the central entity, hence resulting in significant overhead, which necessitates the design of a distributed spectrum allocation algorithm.

In [6], a distributed channel adjustment protocol assigns multiple contiguous channels to each radio such that the frequency bands of two radios transmitting between each other are partially-overlapped. This would decrease spectrum efficiency. Reference [7] enables nodes to communicate on fully-overlapping frequency bands, and presents a joint transport, routing and spectrum sharing (TRSS) optimization algorithm to maximize network utility in a distributed manner. The optimization process consists of two phases: spectral width pre-allocation and link-layer frequency scheduling. In the first phase, a joint spectrum sharing, routing and congestion (JSSRC) algorithm is responsible for computing the spectral widths of links, but does not consider their center frequencies. In the second phase, a timing-window based spectrum reservation (TWSR) scheme is used to approximate the spectral widths obtained in JSSRC. Based on the partition, the JSSRC algorithm can only use a neither sufficient nor necessary condition of interference-free spectrum allocation as its interference constraints, because partial characteristics of frequency bands are not taken into consideration in the first phase. As a result of this, on one hand, the spectral widths obtained in the first phase may be too narrow and thus lead to a sub-optimal policy; on the other hand, the spectral widths computed in the first phase may be too high to be fully

allocated in the second phase. In either case, the TRSS algorithm results in low spectrum utilization.

To improve the spectrum resource utilization in wireless mesh networks with frequency-agile radios, we derive a sufficient and necessary condition for interference-free spectrum allocation under any pairwise interference model. An interference pair refers to two links between which there exists a symmetric or asymmetric interference. We employ a formal method to describe the relationship of the center frequencies and spectral widths of two links in each interference pair. To the best of our knowledge, we are among the first to formalize the interference relationship based on interference pairs in multi-hop frequency-agile radio networks.

Based on a set of rigorous models, we formulate the problem of distributed allocation of center frequencies and spectral widths of links as an optimization problem with an objective to maximize the network utility. Due to the NP-hardness of the spectrum allocation problem [4], there does not exist a polynomial-time optimal solution unless $P = NP$. Therefore, a decomposition method is called for to design and apply efficient heuristic solutions. The most challenging part of the algorithm design is to divide the spectrum allocation problem into appropriate subproblems such that the final result approaches the optimal value. In this paper, we decompose the spectrum allocation problem into two subproblems: (i) determine the relations of center frequencies and (ii) jointly allocate center frequencies and spectral widths. Following that, we design a two-phase interference pair-based distributed spectrum allocation (IPDSA) algorithm. In Phase I, a decentralized negotiation-based frequency hierarchy (NFH) mechanism is proposed for determining the relations of center frequencies of links with a balanced time window function; while in Phase II, a joint center frequency and spectral bandwidth allocation (JFBA) algorithm is designed to solve the network utility optimization problem using a dual decomposition method in a distributed manner, based on the determined relations of center frequencies.

In comparison to the related work in the literature, our work makes the following contributions:

- We propose an accurate formalization of interference-free spectrum allocation in multi-hop frequency-agile radio networks based on sufficient and necessary interference constraints under any pairwise interference model.
- We propose a distributed solution to the joint allocation of center frequencies and spectral widths of links that maximizes the network utility of wireless mesh networks, regardless of the disparity in the available spectrum ranges of radios. This solution consists of a negotiation-based frequency hierarchy mechanism with a balanced time window function and a spectrum allocation algorithm with a provable property of convergence to the global optimality.

The rest of the paper is organized as follows. Section II describes the system model and formulates the network utility optimization problem. In Section III, we introduce the theoretical framework for the design of the IPDSA algorithm. Sections IV and V present the NFH mechanism and the JFBA algorithm, respectively. In Section VI, we evaluate the IPDSA algorithm in terms of spectrum utilization, network performance, and computational overhead. Section VII

provides an overview of the related work. We conclude our work in Section VIII.

II. SYSTEM MODEL AND PROBLEM STATEMENT

We consider a wireless mesh network that consists of a set V of nodes, where are connected via a set L of directed wireless communication links. Note that each node can launch a session to any node except itself. We assume that the transmission power spectral density of each node is determined and the ambient noise remains relatively constant. Such a wireless mesh network is typically represented by a directed network graph $G(V, L)$.

A. Internal Interference Constraints

To address spectrum utilization inefficiency caused by coarse-grained interference constraints, we use interference pairs as a fine granularity to accurately characterize the interference relationship among the internal links in a wireless mesh network. An interference pair refers to an unordered pair of links, between which there exists symmetric or asymmetric interference. The symmetric interference means that a pair of links l and \hat{l} interfere with each other; while the asymmetric interference means that link l causes interference to link \hat{l} , but not vice versa. Specifically, we define a set I of interference pairs, and denote the set of interference links of link l by

$$I(l) = \{\hat{l} \mid \hat{l} \in L, (l, \hat{l}) \in I\}.$$

Based on the notion of interference pairs, we use the center frequency f_l and spectral width b_l of link l to present a sufficient and necessary condition of spectrum allocation without internal interference, and define a spectrum allocation policy as a set $\{(f_l, b_l)\}_{l \in L}$. In a feasible spectrum allocation policy, there is no spectrum overlap between two active links of an interference pair. Thus, we have

$$\begin{cases} 2|f_l - f_{\hat{l}}| \geq b_l + b_{\hat{l}}, \forall (l, \hat{l}) \in I, \\ b_l \geq 0, \forall l \in L \end{cases}, \quad (1)$$

which accurately formulates the spectrum allocation without internal interference. To eliminate the absolute value, we rewrite Eq. (1) as Eq. (1')

$$\begin{cases} 4(f_l - f_{\hat{l}})^2 \geq (b_l + b_{\hat{l}})^2, \forall (l, \hat{l}) \in I \\ b_l \geq 0, \forall l \in L \end{cases} \quad (1')$$

B. External Interference Constraints

Due to the external interference from co-located wireless networks, the available frequency band of a wireless link varies only within the span of vacant spectrum around the link. We denote the upper and lower bounds of the available spectrum of link l as \underline{F}_l and \overline{F}_l , respectively. With the presence of external interference, we limit the frequency band of any link to the range of its available spectrum:

$$\begin{cases} f_l - b_l/2 \geq \underline{F}_l, \forall l \in L \\ f_l + b_l/2 \leq \overline{F}_l, \forall l \in L \end{cases} \quad (2)$$

In sum, a spectrum allocation is interference-free if and only if both Eqs. (1') and (2) hold. Moreover, in Eqs. (1') and (2), the number of variables is $2|L|$, and the number of inequalities is $2|L| + |I| \leq 2|L| + \Delta|L|/2 = O(|L|)$, where the

constant $\Delta = \max_l I(l)$ depends on the density of nodes. Therefore, the number of variables and inequalities in our interference constraints is in the same order as that in the interference constraint in [7], which includes $|L|$ variables and $|L|$ inequalities (See (24)).

C. Flow Conservation Constraints

We denote $r_v^{(u)}$ as the achievable rate of the multi-hop or one-hop flow from the source node v to the destination node u ($v \neq u$), and use $h_l^{(u)}$ to represent the total traffic over link l among all the flows to node u , where link l is a component link. $L^-(v)$ and $L^+(v)$ are the sets of directed links that start and end at node v , respectively. A feasible flow allocation must abide by the flow conservation constraint below:

$$r_v^{(u)} = \sum_{l \in L^-(v)} h_l^{(u)} - \sum_{l \in L^+(v) - L^-(u)} h_l^{(u)}, \forall u \neq v \quad (3)$$

The constraint in (3) states that the achieved flow from node v to node u equals the difference between the aggregated outgoing and incoming traffic of node v to node u . For a given destination node u , the aggregated incoming (or outgoing) traffic of any node v ($v \neq u$) equals the total amount of flows over all the links that end (or start) at node v .

D. Link Capacity Constraints

We use Q_l to denote the quality of link l , which is defined to be the maximum data rate of link l per unit spectral width. Q_l depends on the signal-to-noise ratio SNR_l of link l and the coding efficiency Γ , which indicates the dB gap between uncoded modulation and channel capacity, minus the coding gain. We use l (or \hat{l}) to represent the starting and ending node of link l , respectively, and describe the link capacity constraint as follows

$$\sum_{u \neq l^-} h_l^{(u)} \leq Q_l b_l, \forall l \in L, \quad (4)$$

where $Q_l = \log_2(1 + SNR_l/\Gamma)$, $\forall l \in L$.

The constraint in (4) ensures that the traffic over any link cannot exceed its capacity, according to Shannon's theorem.

E. Network Utility Maximization

We wish to allocate the spectral widths and center frequencies from a global perspective such that the network utility among all the flows is maximized. The network utility can be defined by a series of concave utility functions $U(\cdot)$ [8] in terms of flow rate r .

Based on the above constraints, we formulate the spectrum allocation formulated as a constrained optimization problem:

$$\begin{aligned} & \max_{\sum_v \sum_{u \neq v} U(r_v^{(u)})} \quad (5) \\ \text{s. t. } & \left\{ \begin{array}{l} r_v^{(u)} \leq \sum_{l \in L^-(v)} h_l^{(u)} - \sum_{l \in L^+(v) - L^-(u)} h_l^{(u)}, u \neq v \\ \sum_{u \neq l^-} h_l^{(u)} - Q_l b_l \leq 0 \\ (b_l + b_{\hat{l}})^2 \leq 4(f_l - f_{\hat{l}})^2, (l, \hat{l}) \in I \\ b_l/2 - f_l + \underline{F}_l \leq 0 \\ b_l/2 + f_l - \overline{F}_l \leq 0 \\ \text{All the variables are nonnegative real numbers.} \end{array} \right. \end{aligned}$$

III. AN OVERVIEW OF DISTRIBUTED SPECTRUM ALLOCATION UNDER PAIRWISE INTERFERENCE

Based on the accurate formalization in the previous section, we design an interference pair-based distributed spectrum allocation (IPDSA) algorithm. This section introduces the overall idea of the algorithm design.

The problem formulation in (5) has a non-linear constraint, i.e. the third inequality. Therefore, the feasible region of the problem is not a convex set. For such a non-convex optimization problem, we can only compute its local optimal solution. In order to approximate the global optimal solution, we need to restrict the feasible region of the problem to a convex subset where the optimal or near-optimal solution most likely exists.

Suppose that $(\mathbf{r}^*, \mathbf{h}^*, \mathbf{b}^*, \mathbf{f}^*)$ is a global optimal solution of (5). Then, there exists an ordering $<$ of two links of each interference pair, according to decreasing center frequency f_l^* . If we know the ordering of two links in each interference pair, the non-convex optimization problem (5) can be reduced to a convex optimization problem (6), whose global optimal solution can be solved in polynomial time.

$$\begin{aligned} & \max_{\sum_v \sum_{u \neq v} U(r_v^{(u)})} \quad (6) \\ \text{s. t. } & \left\{ \begin{array}{l} r_v^{(u)} - \sum_{l \in L^-(v)} h_l^{(u)} + \sum_{l \in L^+(v) - L^-(u)} h_l^{(u)} \leq 0, u \neq v \\ \sum_{u \neq l^-} h_l^{(u)} - Q_l b_l \leq 0 \\ b_l + b_{\hat{l}} \leq 2(f_l - f_{\hat{l}}), < l, \hat{l} > \in \tilde{I} \\ b_l/2 - f_l + \underline{F}_l \leq 0 \\ b_l/2 + f_l - \overline{F}_l \leq 0 \\ \text{All the variables are nonnegative real numbers.} \end{array} \right. \end{aligned}$$

In the third constraint of (6), set \tilde{I} represents all the ordered pairs of links, between which there exists interference, and in each ordered pair, the center frequency of the first link is higher than that of the second one.

Unfortunately, we do not know the ordering of each interference pair. Therefore, we firstly need to predetermine the ordering of each interference pair such that it is as similar as possible to the optimal ordering. Then, based on the predetermined ordered pairs, we can obtain the optimal spectrum allocation policy by solving the transformed convex optimization problem (6).

Following this approach, the original spectrum allocation problem is divided into two subproblems. The first subproblem is to determine the relations of center frequencies of links with interference. This is a combinatorial optimization problem. Although it has an extremely high complexity, it only has a relatively minor effect on the final objective value. The second subproblem is to jointly allocate the center frequencies and spectral widths of links, based on certain relations of center frequencies. This is a continuous optimization problem and its optimal solution is computationally obtainable. Hence, such partitioning of the original problem is reasonable and helpful.

Accordingly, the IPDSA algorithm is executed in two phases. In each phase, it is designed in a distributed manner. This does not only benefit parallel computation, but also save

the overhead of global message transmission. In the following two sections, we introduce the NFH mechanism and the JFBA algorithm in detail, which are used in the two phases of the algorithm, respectively.

IV. A MECHANISM FOR DETERMINING THE RELATION OF CENTER FREQUENCIES OF EACH INTERFERENCE PAIR

In this section, we propose the heuristic NFH mechanism which determines the relation of center frequencies of links in each interference pair. The basic idea behind the NFH mechanism is as follows. In an interference pair, (i) if the median of the available spectral range of a link is higher, the center frequency of the link is inclined to be higher; (ii) if the available spectral ranges of two links have the same median, the relation between their center frequencies should depend on their priorities.

For ease of exposition, we define three types of relations between links. (i) We call the relation $<$ as the *prior-to* relation, and $l < \hat{l}$ means that the center frequency of link l is higher than that of link \hat{l} . (ii) Similarly, we call the relation $>$ as the *subsequent-to* relation, and $l > \hat{l}$ means that the center frequency of link l is lower than that of link \hat{l} . (iii) We denote the relation \sim as the *uncertain* relation, and $l \sim \hat{l}$ means that the relation between the center frequencies of links l and \hat{l} is unknown for the moment.

In detail, the NFH mechanism works as the following. Initially, for each interference pair, the relation between two links is uncertain. In the first step, each link l notify the median of its available spectral range to all the links in $I(l)$. After that, two links in each interference pair can agree on their relation type, if their available spectral ranges have different medians. In other words, the link with a higher median of its available spectral range occupies a higher frequency band, in comparison with the other link in the same interference pair.

In the second step, we consider the case that the available spectral ranges of two links in an interference pair have the same median. In the case, we introduce a random contention strategy with a priority in order to maximize spectrum reuse. We define

$$I_{<}(l) = \{\hat{l} \in L | (l, \hat{l}) \in I \wedge l < \hat{l}\} = \{\hat{l} \in L | \langle \hat{l}, l \rangle \in \tilde{I}\}, \forall l$$

$$I_{>}(l) = \{\hat{l} \in L | (l, \hat{l}) \in I \wedge l > \hat{l}\} = \{\hat{l} \in L | \langle l, \hat{l} \rangle \in \tilde{I}\}, \forall l$$

$$I_{\sim}(l) = \{\hat{l} \in L | (l, \hat{l}) \in I \wedge l \sim \hat{l}\}, \forall l$$

Every link l has a weight, $W(l) = |I_{>}(l)| + |I_{<}(l)|$. For an interference pair of uncertain relation, the link with a higher weight has priority to access a higher frequency band.

In the random contention strategy with a priority, the receiver of a link negotiates the frequency relations with others on behalf of the link. Link l randomly picks a time from a specific time window to declare its frequency application to the links in $I_{<}(l)$. If link \hat{l} in $I_{<}(l)$ approves of a frequency application from link l , it immediately returns an acknowledgement message and the relation between links l and \hat{l} is updated to $l < \hat{l}$. When two links in an interference pair simultaneously submit their own frequency applications, a conflict may occur. At the time, the link with a small ID

number succeeds, and the other link notifies corresponding links of the cancellation of its previous application.

The lower and upper bounds of the time window of link l should both be decreasing functions of $W(l)$. We can simply specify that the time window of link l is from $T/[W(l)+1]$ to $T/W(l)$. Here, T is a scalar determined by timing granularity of the network. However, since $W(l)$ and $|I_{<}(l)|$ may both be large at the beginning of the contention process, the contention of many links in a time window with a short length possibly results in conflicts.

Therefore, we intend to design a time window function such that frequency applications are almost uniformly scattered in time range T . Ahead of the second step, the maximum weight of all the links $\Delta_{\geq} = \max_l |W(l)|$ can spread to anywhere in a wireless mesh network via d hops, where d is the diameter of G . Depending on the number of the uncertain relations, the lower and upper bounds of the time window function fall into two categories.

If $\Delta_{\sim} = O(\Delta_{\geq})$, where $\Delta_{\sim} = \max_l |I_{\sim}(l)|$, then

$$\underline{T}(l) = T[\Delta_{\geq}^2 - W^2(l)]/\Delta_{\geq}^2$$

$$\overline{T}(l) = T[\Delta_{\geq}^2 - (W(l) - 1)^2]/\Delta_{\geq}^2$$

If $\Delta_{\sim} = o(\Delta_{\geq})$, then

$$\underline{T}(l) = T[\Delta_{\geq} - W(l)]/\Delta_{\geq}$$

$$\overline{T}(l) = T[\Delta_{\geq} - W(l) + 1]/\Delta_{\geq}$$

According to the above-described NFH mechanism, the relation between center frequencies of two links in each interference pair can be determined in a distributed manner.

V. JOINT CENTER FREQUENCY AND SPECTRAL WIDTH ALLOCATION ALGORITHM BASED ON DUAL DECOMPOSITION

In this section, we present a joint center frequency and spectral bandwidth allocation (JFBA) algorithm to solve the problem (6) in a distributed manner. In the algorithm, only local message exchange is required. Every node independently runs its own work in the algorithm and generates the part of results (r, h, b, f) associated with itself. The algorithm can guarantee that all the parts of results are consistent and tend to the globally optimal solution.

The JFBA algorithm is based on dual decomposition and subgradient projection methods. For convergence of the algorithm, we let $U(\cdot)$ be a twice continuously differentiable, increasing and strictly concave function of $r_v^{(u)}$, such as $\log(r_v^{(u)})$.

Consider the dual problem (7) of the primal problem (6):

$$\min_{\lambda, \mu, \xi, \varpi, \rho \geq 0} D(\lambda, \mu, \xi, \rho, \sigma) \quad (7)$$

with dual function

$$D(\lambda, \mu, \xi, \rho, \sigma) = D_1(\lambda) + D_2(\lambda, \mu) + D_3(\mu, \xi, \rho, \sigma)$$

where

$$D_1(\lambda) = \max_{r_v^{(u)} \geq 0} \sum_v \sum_{u \neq v} [U(r_v^{(u)}) - \lambda_v^{(u)} r_v^{(u)}] \quad (8)$$

$$D_2(\lambda, \mu) = \max_{h_l^{(u)} \geq 0} \sum_v \sum_{u \neq v} \lambda_v^{(u)} \left(\sum_{l \in L^-(v)} h_l^{(u)} - \sum_{l \in L^+(v) - L^-(u)} h_l^{(u)} \right) - \sum_{l \in L} \mu_l \sum_{u \neq l} h_l^{(u)} \quad (9)$$

and

$$D_3(\boldsymbol{\mu}, \boldsymbol{\xi}, \boldsymbol{\rho}, \boldsymbol{\sigma}) = \max_{b_l, f_l \geq 0} \sum_l \mu_l Q_l b_l - \sum_{(l,i) \in I} \xi_{l,i} (b_l + b_l - 2f_l + 2f_i) - \sum_l \rho_l (b_l/2 - f_l + \underline{F}_l) - \sum_l \sigma_l (b_l/2 + f_l - \overline{F}_l) \quad (10)$$

The maximization problem in the dual function can be decomposed into the following three partial dual subproblems. The first subproblem D_1 is an end-to-end congestion control problem which can be solved at the transport layer. The second subproblem D_2 is a link rate scheduling problem which can be solved by routing. The two subproblems are correlated through Lagrange multipliers $\lambda_v^{(u)}$, and are combined as a rate assignment problem. The third subproblem D_3 is a spectrum allocation problem that maximizes the weighted sum of link capacities with Lagrange multipliers μ_l as the weights. Then, the master dual problem D can be treated as an optimization problem regarding Lagrange multipliers μ_l . These separate problems can be solved using distributed iterative algorithms. The JFBA algorithm is distinctive in the spectrum allocation subproblem D_3 .

A. The Rate Assignment Subproblem of D_1 and D_2

For the first subproblem D_1 , it can be seen from (8) that each source node of a session can calculate the optimal rate of the session in a distributed manner as long as local price $\lambda_v^{(u)}$ is known.

$$r_v^{(u)} = [U^{-1}(\lambda_v^{(u)})]^+, u \neq v \quad (11)$$

where $[x]^+ = \max(x, 0)$.

For the second subproblem D_2 , (9) can be converted into (12) by exchanging the order of integrations over u and over l .

$$D_2(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \max_{h_l^{(u)} \geq 0} \sum_{l \in L} \sum_{u \neq l} (\lambda_l^{(u)} - \lambda_{l^+}^{(u)} - \mu_l) h_l^{(u)} \quad (12)$$

Let

$$\lambda_l = \max_{u \neq l} (\lambda_l^{(u)} - \lambda_{l^+}^{(u)}), \forall l$$

$$u_l = \operatorname{argmax}_{u \neq l} (\lambda_l^{(u)} - \lambda_{l^+}^{(u)}), \forall l$$

Then,

$$h_l^{(u)} = \begin{cases} h_l, & u = u_l \\ 0, & u \neq u_l \end{cases}$$

$$h_l = h_l^{(u_l)}$$

The subproblem D_2 can be rewritten as

$$D_2(\boldsymbol{\mu}, \boldsymbol{\lambda}) = \max_{h_l \geq 0} \sum_{l \in L} (\lambda_l - \mu_l) h_l \quad (13)$$

To enhance the stability of the algorithm, we add a small quadratic term $-\tau h_l^2$ ($\tau > 0^+$) in D_2 .

$$D_2(\boldsymbol{\mu}, \boldsymbol{\lambda}) \approx \max_{h_l^{(u)} \geq 0} \sum_{l \in L} (\lambda_l - \mu_l) h_l - \tau h_l^2 \quad (14)$$

In the joint rate assignment subproblem of D_1 and D_2 , Lagrange multipliers $\lambda_v^{(u)}$ can be updated step by step.

$$h_l = \left(\frac{\lambda_l - \mu_l}{2\tau} \right)^+ \quad (15)$$

Hence, each link l can only require local prices λ_l and μ_l to individually compute the optimal link rate.

In addition, Lagrange multipliers $\lambda_v^{(u)}$ can also be updated as (16), only based on the information of node v and its incident links.

$$\lambda_v^{(u)} [m+1] = \left\{ \lambda_v^{(u)} [m] + \beta (r_v^{(u)} - \sum_{l \in L^+(v)-L^+(u)} h_l^{(u)} + \sum_{l \in L^-(v)-L^+(u)} h_l^{(u)}) \right\}^+, u \neq v \quad (16)$$

where β is a positive stepsize.

B. The Spectrum Allocation Subproblem of D_3

For the third subproblem D_3 , the partial dual function in (10) can be equivalently transformed to (17).

$$D_3(\boldsymbol{\mu}, \boldsymbol{\xi}, \boldsymbol{\rho}, \boldsymbol{\sigma}) = \max_{b_l, f_l \geq 0} \sum_{l \in L} (\mu_l Q_l - \rho_l/2 - \sigma_l/2 - \sum_{l \in I(l)} \xi_{l,i} b_l + \sum_{l \in L} (\rho_l - \sigma_l + 2 \sum_{i \in I^+(l)} \xi_{i,l} - 2 \sum_{i \in I^-(l)} \xi_{i,l}) f_l + \sum_{l \in L} (\sigma_l \overline{F}_l - \rho_l \underline{F}_l) \quad (17)$$

Similarly, we add two small quadratic terms $-\tau b_l^2$ and $-\tau f_l^2$ in D_3 . Then, using the subgradient projection method, we derive an iterative approach to solve the regularized partial dual problem as follows.

$$b_l = [(2\mu_l Q_l - \rho_l - \sigma_l - 2 \sum_{i \in I(l)} \xi_{i,l}) / (4\tau)]^+ \quad (18)$$

$$f_l = [(\rho_l - \sigma_l + 2 \sum_{i \in I^+(l)} \xi_{i,l} - 2 \sum_{i \in I^-(l)} \xi_{i,l}) / (2\tau)]^+ \quad (19)$$

$$\xi_{l,i} [m+1] = \{\xi_{l,i} [m] + \gamma (b_l + b_l - 2f_l + 2f_i)\}^+ \quad (20)$$

$$\rho_l [m+1] = \{\rho_l [m] + \gamma (b_l/2 - f_l + \underline{F}_l)\}^+ \quad (21)$$

$$\sigma_l [m+1] = \{\sigma_l [m] + \gamma (b_l/2 + f_l - \overline{F}_l)\}^+ \quad (22)$$

where γ is a positive stepsize.

Note that all computation involved in (18) - (22) needs only local information.

C. The Master Dual Problem

Given \mathbf{h} and \mathbf{b} in D_1 and D_2 , we can solve the master dual problem in (7) by iteratively updating Lagrange multipliers μ_l as (23) according to the information of link l itself.

$$\mu_l [n+1] = \{\mu_l [n] + \alpha (h_l - Q_l b_l)\}^+ \quad (23)$$

where α is a positive stepsize.

The system of (11), (15), (16) and (18) - (23) forms the mathematical base to design the JFBA Algorithm for solving the dual problem in (7). The pseudo code for the JFBA algorithm is provided in Algorithm 1.

D. Convergence and Optimality

The convergence and optimality of the JFBA algorithm is guaranteed by Proposition 1.

Proposition 1. For any sequence of step-length $\{\alpha_n\}$ fulfilling $\alpha_n \rightarrow 0$ and $\sum_{n=1}^{\infty} \alpha_n = \infty$, the JFBA algorithm converges to the globally optimal value.

Proof: Because the objective function is strictly convex and the constraints are affine in (6), the primal problem (6) is a strictly convex optimization problem. Therefore, the primal problem (6) has a unique locally optimal solution, which is the globally optimal solution; and the optimal value of the dual problem (7) equals the optimal value of the primal problem (6).

With Lagrange decomposition methods, the domain of the dual function (7) is convex and the dual function (7) is concave over its domain. According to Proposition 4.2 in [9], for any sequence of step-length $\{\alpha_n\}$ fulfilling $\alpha_n \rightarrow 0$ and $\sum_{n=1}^{\infty} \alpha_n = \infty$, sequence $\{\mu_n\}$ in subgradient projection methods includes the optimal solution. Thus, the JFBA algorithm converges to the globally optimal value. \square

For ease of asynchronous computation, we consider the case that α is a fixed step length. If we assume the norm of the subgradients of the dual function is bounded, the convergence of the JFBA algorithm is guaranteed by the following proposition.

Proposition 2. If $\|\mathbf{h}(\boldsymbol{\mu}[t]) - \mathbf{Q}^T \mathbf{b}(\boldsymbol{\mu}[t])\| \leq M, \forall t$ holds, the network utility generated by the iterative computation of the JFBA algorithm converges statistically to within $\alpha M^2/2$ of the optimal value, where M is a constant.

We skip the proof for Proposition 2 due to space limitations.

Algorithm 1: The JFBA Algorithm

Input: $G(V, L), r(0)$
Output: $\mathbf{f}, \mathbf{b}, \mathbf{h}, \mathbf{r}$

- 1 Initialize $(\mathbf{f}, \mathbf{b}, \mathbf{h}, \mathbf{r}, \lambda, \mu, \xi, \rho, \sigma)$
- 2 do
- 3 do
- 4 $r_v^{(u)} \leftarrow [U^{-1}(\lambda_v^{(u)})]^+$
- 5 $\lambda_l \leftarrow \max_{u \neq l} (\lambda_l^{(u)} - \lambda_l^{(u)}); u_l \leftarrow \operatorname{argmax}_{u \neq l} (\lambda_l^{(u)} - \lambda_l^{(u)})$
- 6 $h_l \leftarrow h_l^{(u_l)} \leftarrow \left(\frac{\lambda_l - \mu_l}{2\tau}\right)^+; h_l^{(u)} \leftarrow 0, u \neq u_l$
- 7 $\lambda_v^{(u)} \leftarrow [\lambda_v^{(u)} + \beta(r_v^{(u)} - \sum_{l \in L^+(v)-L^+(u)} h_l^{(u)} + \sum_{l \in L^-(v)-L^+(u)} h_l^{(u)})]^+$
- 8 while the variation of $\lambda > \delta_1$
- 9 do
- 10 $b_l \leftarrow [(2\mu_l Q_l - \rho_l - \sigma_l - 2 \sum_{l \in I(l)} \xi_{l,l}) / (4\tau)]^+$
- 11 $f_l \leftarrow [(\rho_l - \sigma_l + 2 \sum_{l \in I_>(l)} \xi_{l,l} - 2 \sum_{l \in I_<(l)} \xi_{l,l}) / (2\tau)]^+$
- 12 $\xi_{l,l} \leftarrow [\xi_{l,l} + \gamma(b_l + b_l - 2f_l + 2f_l)]^+$
- 13 $\rho_l \leftarrow [\rho_l + \gamma(b_l/2 - f_l + \bar{F}_l)]^+$
- 14 $\sigma_l \leftarrow [\sigma_l + \gamma(b_l/2 + f_l - \bar{F}_l)]^+$
- 15 while the variation of ξ, ρ and $\sigma > \delta_2$
- 16 $\mu_l \leftarrow \{\mu_l + \alpha(h_l - Q_l b_l)\}^+$
- 17 while the variation of $\mu > \delta_3$

E. Understanding the JFBA Algorithm

Let us take a closer look at the JFBA algorithm to gain some insight on its physical meaning. The key is to understand Lagrange multipliers ξ, ρ, σ, μ and λ .

Lagrange multiplier $\xi_{l,\hat{l}}$ can be interpreted as the spectrum contention price of links l and \hat{l} in the interior of a wireless mesh network. The price is what link l needs to be paid for occupying a unit of spectrum of link \hat{l} . Apparently, there is a tradeoff between local and global spectrum utilization maximization. Lagrange multipliers $\xi_{l,\hat{l}}$ seem as messengers to passing this tradeoff information among links. When the frequency bands of links l and \hat{l} overlap with each other, the JFBA algorithm increases their contention price $\xi_{l,\hat{l}}$ in (20) to

prevent them from taking more spectrum from the other side. When there is still unallocated available spectrum between links l and \hat{l} , contention price $\xi_{l,\hat{l}}$ is reduced so that the two links are encouraged to use this unallocated spectrum.

Lagrange multipliers ρ_l and σ_l can be viewed as the spectrum access prices of link l due to the existence of external interference. It is considered as the price that needs to be paid by link l for using a unit of unavailable spectrum. In (21) and (22), the JFBA algorithm increases access price ρ_l or σ_l of link l as a penalty when its frequency band is beyond the lower or upper bound of its available spectrum. When there is still unallocated available spectrum for link l , access prices ρ_l and σ_l are decreased to promote the utilization of this unallocated spectrum.

Lagrange multipliers u_l and $\lambda_v^{(u)}$ can be regarded as the link price of the link l , and the path price from node v to node u , respectively. With respect to u_l and $\lambda_v^{(u)}$, the reader can refer to [7] and [10] for more detailed explanation.

VI. PERFORMANCE EVALUATION

In this section, we study performance of the IPDSA algorithm and evaluate its effectiveness in terms of spectrum utilization, network performance and computational overhead. We choose the TRSS algorithm [7] for comparative evaluation, and investigate the IPDSA algorithm in three types of network scenarios: (i) All nodes are in a regular 7×7 grid topology, and all links have uniform available spectrum. (ii) All nodes are randomly dispersed in a square area of $300 \times 300 \text{ m}^2$ to form a fully connected topology, and all links have uniform available spectrum. (iii) All nodes are also randomly dispersed in a square area of $300 \times 300 \text{ m}^2$. However, the available spectrums of all links are not uniform, and each one is a random frequency band over a range of spectrum. The total spectral width is set to 240 MHz. We assume the maximum transmission distance is 90 m and the maximum interference distance is 135 m. We adopt the similar parameters of link capacity to [5] (from 0.3 bps/Hz to 2.7 bps/Hz like IEEE 802.11a). In the regular 7×7 grid topology, we select 14, 8, and 6 cross sessions in the identical or opposite directions, shown as Fig. 1. In random topologies, there is a session between any pair of nodes.

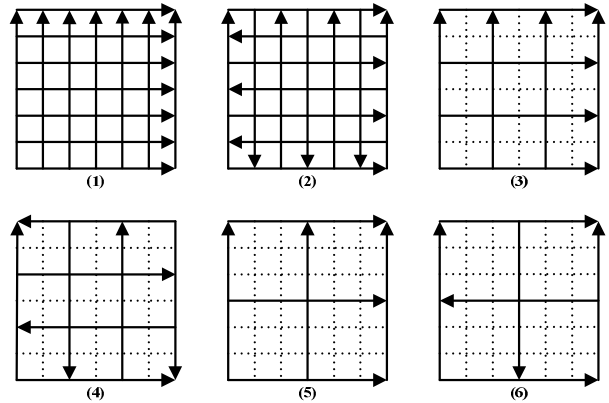
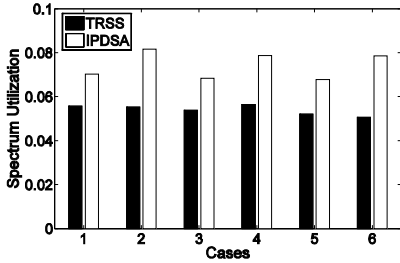
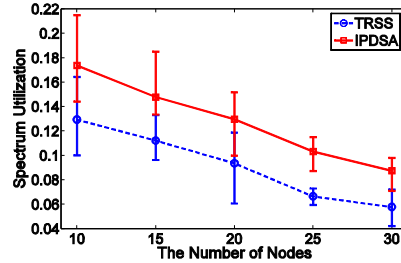


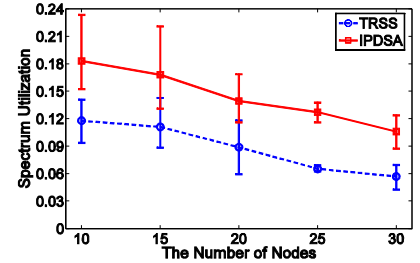
Figure 1. Regular topology and session pairs



(a) Regular topology and uniform spectrum

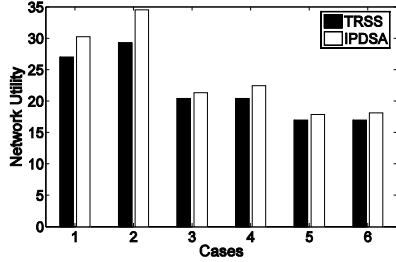


(b) Random topology and uniform spectrum

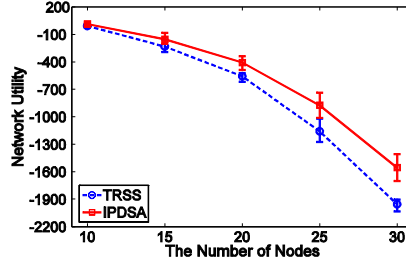


(c) Random topology and un-uniform spectrum

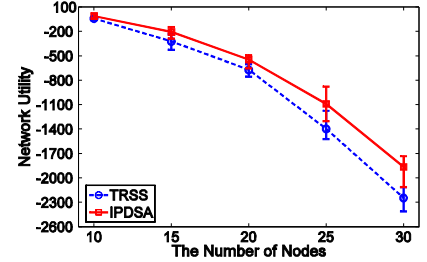
Figure 2. Spectrum utilization in various scenarios



(a) Regular topology and uniform spectrum

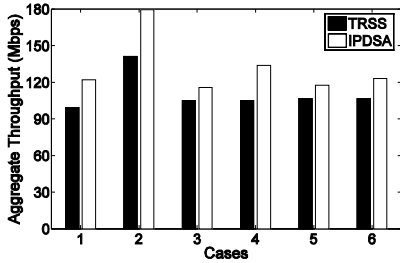


(b) Random topology and uniform spectrum

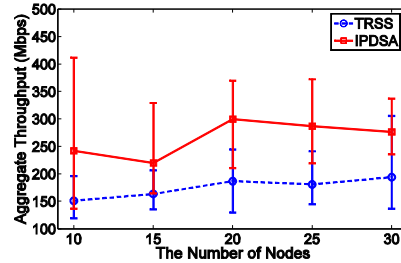


(c) Random topology and un-uniform spectrum

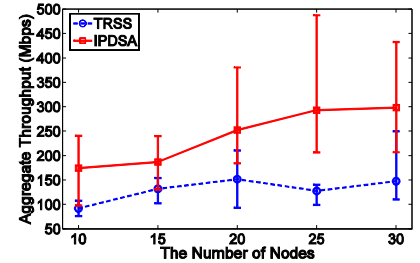
Figure 3. Network utility in various scenarios



(a) Regular topology and uniform spectrum



(b) Random topology and uniform spectrum



(c) Random topology and un-uniform spectrum

Figure 4. Aggregate throughput in various scenarios

A. Spectrum Utilization

We measure spectrum utilization by an average of spectrum utilization at all the nodes, and spectrum utilization at a node is the ratio of the width of frequency band on which the node receives data to the total width of available spectrum at the node. Then, we evaluate the spectrum utilization of IPDSA and TRSS algorithms in the above-mentioned three types of scenarios, and observe the effect of node density on spectrum utilization. Fig. 2 reveals that the IPDSA algorithm respectively provides 26.0% to 55.3%, 31.8% to 55.4%, and 51.4% to 95.2% higher spectrum utilization in the three types of scenarios, compared with the TRSS algorithm. In addition, with the number of nodes increasing, the amount of spectrum utilized by a single node descends. In Fig. 2(b), (c), each point is the average result of five inputs, and an error bar indicates the maximum and minimum in a group of results. (The following is the same.)

B. Network Performance

We examine the network performance of IPDSA and TRSS algorithms in terms of network utility and aggregate throughput.

Using $\log(\cdot)$ as utility function $U(\cdot)$, we obtain their network utility and aggregate throughput under the proportional fairness. Since the rate of a session may be more or less than 1 Mbps, network utility is positive or negative and thus we focus on the relative relation. Figure 3 plots the network utility in various scenarios, and illustrates that the IPDSA algorithm overperforms the TRSS algorithm in all evaluation scenarios. Figure 4 demonstrates that in comparison with the TRSS algorithm, the IPDSA algorithm respectively increases aggregate throughput by 10.1% to 27.2%, 34.6% to 60.6%, and 42.1% to 132.2% in the three types of scenarios, because it effectively exploits spectrum resources.

C. Computational Overhead

Figure 5 displays the ratios of the numbers of iterations of the IPDSA algorithm to the TRSS algorithm in the second and third scenarios. From the figure, we can observe that although the ratios in the two scenarios both increase with the number of nodes with fluctuations (such as the decline of the ‘‘Uniform Available Spectrum’’ curve from 25 to 30 nodes), the number of iterations of the IPDSA algorithm is of the same order of

magnitude as the TRSS algorithm. A moderate computational overhead can be tolerated to achieve dramatically improved network performance for a long term.

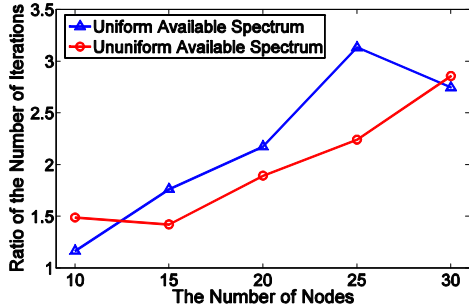


Figure 5. Ratios of the numbers of iterations of IPDSA to TRSS

VII. RELATED WORK

In this section, we review existing research efforts that are most pertinent to our work.

A. Interference Constraints for Multi-hop Wireless Networks

We investigate various interference constraints used in existing spectrum allocation algorithms for multi-hop wireless networks. Interference constraints in multi-hop wireless networks have not been proposed before based on pairs of interference links, although an interference constraint in WLANs [11] is considered based on pairs of neighboring APs. Generally, the existing algorithms can be divided into four categories with respect to interference constraints.

1) *No Force Constraints*: Some algorithms or protocols intend to assign interference-free or low interference channels to nodes or links, but may still suffer from interferences. In [12], the algorithms based on s-disjunct superimposed channel codewords only achieve interference-free channel assignment under certain conditions. In the ROMA protocol [13], channel sequences are chosen only for the purpose of reducing inter-path interference. In our work, the interference constraints ensure an interference-free solution.

2) *Constraints on Network or Interference Graphs*: The approach of depicting the interference relationship in network graphs is only appropriate to specific interference models, such as modeling under duplexing constraints [14], primary interference constraints [15] or secondary interference constraints [16]. However, it cannot be used in measurement driven or distance interference models. Vertex coloring in interference graphs [17] is only suitable for spectrum allocation problems of fixed channelization, but cannot adequately represent variable width spectrum allocation problems. In this paper, our interference constraints can be used under the arbitrary pairwise interference model to solve variable width spectrum allocation problems.

3) *Clique-based Constraints*: For each clique in the interference graph, the sum of the indicator variables of all the vertices (i.e. a link) is no more than one. This clique constraint [18] is a sufficient and necessary condition. However, for the convenience of computation, the constraint needs a relaxation of integer variables to continuous variables in terms of link flows. As a result, the constraint becomes a necessary condition that in each clique, the sum of air-time fractions of

all the links is no more than a constant, which is set to one. Thus, the resulting flow vectors are not always schedulable. To obtain a feasible result, [19] has to restrict the constant to a smaller value and thus deviates from the optimal solution. In this paper, our interference constraints are based on interference pairs at a finer-grained level than clique-based constraints.

4) *Link Centric Constraints*: Most link centric constraints reflect interference relationship by limiting the fraction of air time, which is a ratio of the flow rate to the capacity of a link in a channel. A few link centric constraints use a 0/1 binary variable to indicate whether a link is active in a channel. These constraints fall in the following three categories.

- *Sufficient Conditions*: There are two types of conditions. One type is that the sum of air-time fractions of a link and its interfering and interfered links is no more than a constant, which is set to one [3], [20] - [22]. The algorithms based on the constraint increase the constant step by step on trial to search for a better feasible solution. The other type requires that the sum of indicator variables of a link and its interfering links is no more than 1 [5]. The algorithm based on the constraint needs to approximately solve an integer linear programming problem.

- *Necessary Conditions*: Based on the interference degree [23], the independence number and the induction number [24], three necessary conditions are given correspondingly, and are used to design approximate algorithms.

- *Neither Sufficient nor Necessary Conditions*: For asymmetric interference, a constraint that the sum of air-time fractions of a link and its interfering links is not more than 1 is presented in [25] for a tradeoff between the feasibility and the optimality.

Additionally, a link centric constraint is proposed based on the spectral bandwidth of links [7]. The constraint specifies that the sum of spectral widths of a link and its interfering links does not exceed the total available spectral width of the link:

$$b_l + \sum_{i \in E_l'} b_i \leq \tilde{b}_l, l \in L, \quad (24)$$

where E_l' denotes the set of all links that can cause interference to link l ; b_l and \tilde{b}_l correspond to the allocated spectral width of link l and the spectral spans of the available vacant space for link l , respectively. Since the constraint (1) is neither sufficient nor necessary, the algorithm obeying it may underestimate or overestimate the spectral widths of links. In this paper, our interference constraints are a sufficient and necessary condition of interference-free spectrum allocation.

B. Spectrum Allocation Algorithms Based on Iterative Convergence

Reference [22] utilizes the generalized Benders decomposition techniques to decouple combinatorial constraints and continuous constraints of a robust channel assignment problem. Nevertheless, the interference margin and Lagrange multipliers need to be reported to a central server to solve the master problem. In this paper, we aim at designing a fully distributed algorithm.

A distributed learning automata-based channel allocation protocol (LCAP) is presented in [26]. Each node independently and iteratively learns channel allocation using a probabilistic adaptation algorithm for efficient channel utilization. However, the convergence is not proved and the algorithm may lead to an instable state, which is avoided in our algorithm.

Based on approximate dual decomposition techniques, [19] proposes joint channel allocation, interface assignment and MAC design. However, the algorithm can only be used in spectrum allocation of fixed channelization. Hence, we pay attention to variable width spectrum allocation.

A joint Transport, Routing and Spectrum Sharing optimization algorithm [7] is designed based on the dual decomposition and subgradient projection method. Due to disjunction of spectrum allocation into two phases of spectral width assignment and frequency scheduling, the algorithm can only use coarse-grained interference constraints in the first phase and thus leads to low spectrum efficiency. In comparison with this algorithm, our proposed algorithm can utilize spectrum more efficiently as shown in Section VI.

VIII. CONCLUSION

In this paper, we employed interference pairs as a fine granularity of interference-free spectrum allocation, based on which, we rigorously modeled the interference constraints at a fine-grained level, jointly using the center frequencies and spectral widths of links. Based on an accurate formalization and appropriate partitioning of the interference-free spectrum allocation problem, we developed a two-phase interference pair-based distributed spectrum allocation algorithm. In Phase I, a negotiation-based frequency hierarchy mechanism was employed to heuristically determine the relation of center frequencies in each interference pair, using a balanced time window function to reduce the conflicts in the negotiation process. In Phase II, a dual decomposition-based spectrum allocation algorithm was designed with a provable property of convergence to the joint optimal allocation of center frequencies and spectral widths. Extensive simulation-based performance evaluation illustrated that at the cost of an acceptable computational overhead, the IPDSA algorithm provides significant performance improvements in terms of spectrum utilization, and hence increases network utility and aggregate throughput. In the future, we plan to use a Quasi-Newton algorithm to reduce the number of iterations of IPDSA for lower computational and communication overheads.

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